Representation and reasoning with disjunctive temporal constraints

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Abstract

We show the expressiveness provided by a Labelled point-based metric model for specifying and reasoning about complex disjunctive temporal constraints. The model allows us to manage disjunctive assertions, conjunctive and hypothetical queries, and one-to-many constraints. Additionally, it becomes an adequate support for reasoning on costs associated to constraints.

1. Introduction

A Temporal Constraint Satisfaction Problem (TCSP) is a particular class of CSP problem where variables represent times and constraints represent sets of allowed temporal relations between them. It requires a temporal reasoning model which is made up by (i) a temporal algebra which determines the expressiveness of the temporal model, and (ii) temporal reasoning algorithms [5]. A clear trade-off between these two elements exist. The management of disjunctive constraints allows a higher level of expressiveness, but it implies a higher computational cost [6].

The main classical disjunctive temporal models are point-based [12] ,interval-based [3], metric (quantitative) point-based models [7]. Moreover, some efforts have been made to integrate qualitative and quantitative temporal information, as [9] among others. However, many application domains (scheduling, causal reasoning, etc.) need to manage disjunctive assertions of temporal constraints, as well as conjunctive and hypothetical queries, one-to-many constraints, etc. This gives rise to the need to manage non-binary constraints.

Although there are some models for non-binary CSP [10], [8], we intend to show the high expressiveness provided by the Labelled TCSP model [4], which allows us to specify and reason about a great variety of constraints, such as disjunctive assertions, other complex and non-binary time-point constraints, and costs associated to constraints. This model is based on a labelled point-based disjunctive metric temporal algebra, which gives rise to a labelled-TCN (LTCN)

2. Temporal model of disjunctive constraints

In [4], a new-labelled temporal algebra has been described, whose main elements are:

Labelled disjunctive metric constraints (l_{ij}). The general form of a constraint l_{ij} is:

$(t_i\{([d^-_{ij.1}, d^+_{ij.1}], \{lb_{ij.1}\}), ..., ([d^-_{ij.n}, d^+_{ij.n}], \{lb_{ij.n}\})\}t_j)$[1], with $d^-_{ij.k} \leq d^+_{ij.k}$, which means:

$(t_j - t_i \leq [d^-_{ij.1}, d^+_{ij.1}]) \vee ... \vee (t_j - t_i \leq [d^-_{ij.n}, d^+_{ij.n}])$. Each $[d^-_{ij.k}, d^+_{ij.k}]$ denotes an *elemental* or *canonical* constraint $ec_{ij.k}$.

Label sets associated to *canonical constraints* $(\{lb_{ij.k}\})$. Each $ec_{ij.k}$, when it is asserted, has associated a label set $\{lb_{ij.k}\}$ with $| \{lb_{ij.k}\} |= 1$ that identi£es it. We name the tuple $lec_{ij.k} = (ec_{ij.k}, \{lb_{ij.k}\})$ as *labelled canonical* constraint. Each $lb_{ij.k}$ can be considered as a unique symbol. It uses the special label 'R0' to denote that an input constraint l_{ij} has only one disjunct. Each $lb_{ij.k} \in \{lb_{ij.k}\}$ of a derived constraint provides information about which input ec_{ij} set has been asserted.

Sets of *inconsistent canonical constraints* (*I-L-Sets*). An (*I-L-Set*) is a set of labels $\{lb_{ij.k}\}$ that represents a set of overall inconsistent canonical constraints. That is,

[1] we will use also the form $(t_i\{[d^-_{ij.1}, d^+_{ij.1}]_{\{lb_{ij.1}\}}, ..., [d^-_{ij.n}, d^+_{ij.n}]_{\{lb_{ij.n}\}}\}t_j)$

they cannot all simultaneously hold. Canonical constraints $lec_{ij.k} \in l_{ij}$ are pairwise disjoint. Thus, each 2-length set of labels from each pair of $lec_{ij.k}$ is considered an *I-L-Set*, and is added to the super-set named *Inconsistence-Set (I-set)*.

This model uses a LTCN (Labelled Temporal Constraint Network) based representation, and includes the special temporal points T0 and TF to indicate the beginning and ending of the world. The reasoning algorithms guarantee the consistency and obtain a minimal and global consistent LTCN. These are the *updating process*, and a *total closure process* that infers new constraints from those explicitly asserted. Moreover, we have introduced a mixed *Closure-CSP* algorithm that obtain one or several solutions [2].

3. Complex and non-binary constraints

By reasoning on labelled disjunctive constraints, associated label lists and *I-L-Sets*, the TCSP model offers the capability of representing and managing non-binary disjunctive constraints. Particularly, logical relations among canonical constraints of different edges $(lec_{ij.x} \in lc_{ij}, lec_{kl.y} \in lc_{kl})$ can be speci£ed. This feature will allow us to manage logical expressions of constraints between different pairs of nodes both in assertion and retrieval processes.

3.1. Reasoning of sets of constraints and their consequences

Let us show a simple ¤ow-shop scheduling example to illustrate some features about the retrieval of logical expressions from constraints and the consequences of such expressions. Three jobs $\{J_1, J_2, J_3\}$ share three resources $\{M_1, M_2, M_3\}$ in a given order (Table 1). The use of M_j by J_i gives rise to the operation O_{ij}.

Table 1. A scheduling example.

	rt	M_1	M_2	M_3	dt
J_1	0 (rt_1)	10' (O_{11})	10' (O_{12})	10' (O_{13})	40' (dt_1)
J_2	0 (rt_2)	10' (O_{21})	10' (O_{22})	10' (O_{23})	50' (dt_2)
J_3	0 (rt_3)	10' (O_{31})	10' (O_{32})	10' (O_{33})	60' (dt_3)

rt = Ready-time; dt = Due-time

Each operation O_{ij} can be represented by the temporal points O^-_{ij} and O^+_{ij}, that denote the beginning and ending points of O_{ij}. We can specify this scheduling problem as the following sets of disjunctive temporal metric constraints:

Ready-time: (T0 $\{[rt_i, \infty[_{\{R_0\}}\}$ O^-_{i1}) ∀ i=1..3

Flow-shop: (O^+_{ij} $\{[0, \infty[_{\{R_0\}}\}$ $O^-_{i(j+1)}$) ∀ i=1..3, j=1,2

Due-time: (T0 $\{[0, (dt_j)[_{\{R_0\}}\}$ O^-_{i3}) ∀ i=1..3

Disjunctive:

(O^+_{ij} $\{[1, \infty[_{\{R_{ij,(i+k)j}\}},]-\infty, 0]_{\{R_{ji,j(i+k)}\}}\}$ $O^-_{(i+k)j}$) ∀ i=1..3, ∀ j=1,2,3, ∀ k=1,2, with $O^+_{ij}, O^-_{(i+k)j} \in O$ and $\{R_a, R_b\}$ is an *I-L-Set*. Each disjunctive label $R_{a,b}$ indicates a possible order between operations. For example $R_{12,22}$ re¤ects the fact that O_{12} is scheduled before O_{22}.

Given this constraint set, we apply the corresponding reasoning algorithms outlined in section 2, and we obtain the minimal LTCN. In addition, we obtain an *Inconsistence-Set (I-Set)* that contain sets of *I-L-sets*. This allows us to formulate questions about the feasibility of a conjunctive sets of constraints between different pairs of time points. As an example, suppose that the *I-set* associated to constraint sets of table 1 contains the *I-L-set* $\{R_0, R_{11,12}, R_{22,21}\}$. We can formulate questions such as: *"It is possible for O_{11} to be before O_{12} and for O_{21} to be after O_{22}?"*: Under the common problem context (R_0), the answer is no, since the set of labels associate to these disjunctions $\{R_0, R_{11,12}, R_{22,21}\}$ is an *I-L-set*

These kind of questions can be solved without propagating their effects to all of the LTCN, since a global consistent LTCN is obtained [7], [4]. Moreover, partial solutions can be assembled without having to propagate the partial instantiation to all LTCN. Thus, we can formulate hypothetical queries on the LTCN: *'What happens if ....?'*. For example: we ask if O_{22} can meet with O_{21}, and if O_{12} can meet with O_{11} due to some optimal criteria. Suppose that we have in the minimal LTCN: (O^+_{22} $\{..., [0, 0]_{\{R_0, R_2, R_5, ...\}}, ...\}$ O^-_{21}), (O^+_{12} $\{..., [0, 0]_{\{R_0, R_2, R_3, ...\}}, ...\}$ O^-_{11}). If the union set of labels $\{R_0, R_2, R_3, R_5, ...\}$ is not an *I-L-set*, then we know that both of these choices can hold. In this case, we know also that these choices about the operations (O_{22}, O_{21}) and (O_{12}, O_{11}) imply that the associated disjunctions to the corresponding union-set of labels should also hold.

3.2. Reasoning on complex time-point constraints

A speci£c kind of non-binary constraint (*disjunctive one-to-many constraints*) disjunctively restricts the temporal occurrence of a time-point with other time-points. For instance, a time point can be temporally restricted to the maximum/ minimum temporal occurrence of a set of time-points. These complex constraints are useful in scheduling problems where the ending time-point of each order should be associated to the ending time-point of the last task in the order. Moreover, these constraints can also be useful in *'reasoning about change'* processes. In a typical causal relation $(C1, C2, C3 \rightarrow E)$, the effect *E* holds while all causes $\{C1, C2, C3\}$ hold. Thus, the concluded effect *E* should be temporally constrained to the overlapping temporal interval where all causes $\{C1, C2, C3\}$ hold. That is, $E^- = max$ $(C1^-, C2^-, C3^-)$, $E^+ = min(C1^+, C2^+, C3^+)$. Constraints of this kind, which also appear with non-disjunctive constraints, cannot be managed by usual models so that *ad hoc* procedures are needed [11].

Thus, by using labelled temporal constraints, *"the time-point t_f is at the maximum occurrence of the set of time points* $\{t_1, t_2, .., t_n\}$" can be represented as:

$(max(t_1, t_2, .., t_n)\{[0, 0]\}t_f) \equiv$

$\forall t_i \in \{t_1, .., t_n\} : (t_i\{[0, 0]_{\{R_{Ai}\}}, [1, \infty]_{\{R_{Bi}\}}\}t_f)$

and $\{R_{B1}, R_{B2}, .., R_{Bn}\}$ is an *I-L-Set*. The *I-L-Set* $\{R_{B1}, R_{B2}, .., R_{Bn}\}$ speci£es that the time point t_f has the constraint [0,0] with, at least, a time point t_i. Moreover, the constraint (t_i $\{[0, 0]\}t_f$) can only exist between t_f and those time points t_i that also allow the constraint [0,0] among them. Afterwards, when all time points become fully constrained on the time line, t_f becomes constrained with only the maximum time-points t_{max}: (t_{max} {[0,0]} t_f). In another case, the constraint (t_{max} $\{[1, \infty]\}_{\{R_{Bi}\}}\}$ t_f) would fail.

Finally, given the causal relation $I_1, I_2, ..., I_n \rightarrow I_0$, the following constraints:

$(max(I_1^-, I_2^-, ..., I_n^-)\{[0, 0]\}I_0^-)$,

$(min(I_1^+, I_2^+, ..., I_n^+)\{[0, 0]\}I_0^+)$,

restrict the interval I_0 to the temporal overlapping of intervals $\{I_1, I_2, ..., I_n\}$. Moreover, the constraint ($I_0^-\{[1\infty]\}I_0^+$) implies that I_0 has a non-null duration, so that forces $\{I_1, I_2, ..., I_n\}$ to overlap, which is an usual requirement in causal chaining processes.

3.3. Cost constraints

The labels of the model allows us to incorporate several additional information that can be managed in an integrated manner with the reasoning algorithms. As an example of this feature, we show how specify cost associated to resources required to carry out the corresponding actions depending on the time in which the actions are carried out. The form of a cost constraint would be: *"The resource r_k have a cost associated of x if it is used between the temporal points t_1 and t_2"*.

We denote the cost of use of the resource r_k by means the labelled temporal interval $[t_1, t_2]_{R_{ck}}$. In order to incorporate it into the network, we add the constraint $(t0\{[-\infty, t_1 - 1]_{R_a}, [t_1, t_2]_{R_{ck}}, [t_2 + 1, \infty]_{R_b}\}TF)$, in which R_a and R_b represent zero cost value. Thus, heuristics applied can use label R_{ck} in order to calculate the corresponding operation costs. This will allow us to use the same resolution method to solve the CSP problem, with or without associated costs to actions.

4 Applications

Due to the reasoning algorithm outlined in section 4, we are able to apply it both in the framework of temporal reasoning (for example causal reasoning) and CSP problems [1]. For example, this model has been investigated under several types of scheduling problems, and it has been capable of specifying sets of constraints not contemplated in previous approximations, such as setup and maintenance periods of the resources, several work ¤ows, considering the cost of use of resources, and other types of scheduling problems such as production lots [1].

5. Conclusions

We have shown the expressiveness of a Labelled TCSP model in order to specify and reason about disjunctive temporal constraints. This model allows us to represent and manage non-binary disjunctive constraints both in assertion and retrieval processes, to perform hypothetical queries and assemble partial solutions without having to propagate the partial instantiation to all LTCN. This feature is important in order to analyse and optimize feasible solutions. Moreover, the model allows us to manage cost of constraints (i.e.: cost of use of resources, that can be managed in an integrated way.

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