Learning and Perception

Facultad de Informática
Universidad Politécnica de Valencia

Theme 2:
Preprocessing and Feature Extraction

Enrique Vidal
Alfons Juan
Roberto Paredes
C. Martínez

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Summary

1. Introduction: a classic example ▶ 1
2. Image acquisition and preprocessing ▶ 8
3. A simple task: Optical Character Recognition (OCR) ▶ 14
4. Feature extraction: “direct” methods ▶ 21
5. Feature Extraction: “local” methods ▶ 29
6. Primitive extraction: chain codes ▶ 35
7. Another example: chromosome recognition ▶ 39
8. Acquisition and Preprocessing of Acoustic Signals ▶ 43
9. Feature Extraction: Vector Quantification ▶ 50
A CLASSIC EXAMPLE: CLASSIFICATION OF IRIS FLOWERS

- A typical “academic” problem, introduced by Fisher in 1936.

- The task is to classify correctly flower specimens of the iris family using the information of their petal and sepal sizes.

- The data set contains the measures of 150 specimens of three subclasses: Setosa, Versicolor and Virginica.

- It is frequently used as an example task for comparing the performance and possibilities of different methods of data analysis and pattern recognition.
SPECIMENS OF THREE VARIETIES OF IRIS FLOWERS

Virginica

Versicolor

Setosa
SPECIMENS OF THREE VARIETIES OF IRIS FLOWERS (cont.)

Setosa

Versicolor

Virginica
FEATURE EXTRACTION: desirable properties

- *Continuity and discriminative capacity*: The similarity between the representations of two objects must be in direct correspondance with the similarity with which the objects are *perceived*:

  objects of the same class should have similar representations, while objects of different classes should have different representations.

- *Invariance under common transformations and distortions*: Different instances of the same object should have similar representations.
Feature Vectors (4 components)  
extracted from 3 flower classes from the family of irises

Sepal length, sepal width, petal length, petal width  
(in centimeters)

<table>
<thead>
<tr>
<th>Iris Setosa</th>
<th>Iris Versicolor</th>
<th>Iris Virginica</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 3.5 1.4 0.2</td>
<td>7.0 3.2 4.7 1.4</td>
<td>6.3 3.3 6.0 2.5</td>
</tr>
<tr>
<td>4.9 3.0 1.4 0.2</td>
<td>6.4 3.2 4.5 1.5</td>
<td>5.8 2.7 5.1 1.9</td>
</tr>
<tr>
<td>4.7 3.2 1.3 0.2</td>
<td>6.9 3.1 4.9 1.5</td>
<td>7.1 3.0 5.9 2.1</td>
</tr>
<tr>
<td>4.6 3.1 1.5 0.2</td>
<td>5.5 2.3 4.0 1.3</td>
<td>6.3 2.9 5.6 1.8</td>
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<td>7.2 3.6 6.1 2.5</td>
</tr>
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...  ...  ...
IRIS FLOWERS: BIDIMENSIONAL REPRESENTATION

- 'virginica'
- 'versicolor'
- 'setosa'

Petal Width vs. Petal Length chart.
PÁGINA INTENCIONADAMENTE EN BLANCO
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9 Feature Extraction: Vector Quantification ▶ 50
IMAGE ANALYSIS

- Images constitute the support of one of the most important senses in the human perceptive system.

- Image analysis has countless applications, such as optical character recognition (OCR), fingerprint recognition, face identification, element recognition in biomedical images (p.e., chromosomes), remote sensing, scene analysis, robotics, etc.

- There are specific problems associated to the big amount of information that often must be processed.

- Image processing has a *per se* interest, independently of its use for recognition.
IMAGE ACQUISITION

- **Image:** A function $f(x, y)$ representing the *brightness* of each point of *coordinates* $(x, y)$.

- **Digital Image:** Function $f(x, y)$ discretized in *domain* (coordinates) and *range* (brightness)

- **Sampling:** Discretization of the *domain*; **Quantification:** Discretization of the *range*; 
  *Space Resolution:* pixels/inch;  *Levels or colours:* bits/pixel

Light intensity (brightness) measured in space coordinates $(x,y)$
EXAMPLE OF AN ACQUISITION DEVICE

Surface scanner

Optical resolution: 600 dpi
Scanning modes:
  1 bit (black and white)
  4 bits (16 grey levels)
  8 bits (256 grey levels)
  24 bits (16,7 million colors)

With 1000 dpi and 8 bits:
827 pixels/line x 1169 lines x 1 byte/pixel
= 1 Mbyte
LIMITS OF SPACE RESOLUTION

Original image and images sampled with a resolution or space frequency of 128, 64 and 32 dots per inch (dpi):

Decreasing resolution implies losing bigger details.
ALIASING, NYQUIST FREQUENCY and the SAMPLING THEOREM

- Original image (1x1 inches)
- Space frequency of the finest periodicities ("details") of interest: $P \approx 50\text{dpi}$
- Digital Images: A:140dpi, B:70dpi, C:44dpi, D:22dpi

- **Nyquist Frequency (Sampling Theorem):**
  If $P$ is the space frequency of the finest periodicity in an image and $F$ is the sampling resolution, then the original image can only be exactly reproduced if:

$$F > 2P$$
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A SIMPLE EXAMPLE TASK IN IMAGE RECOGNITION:
Optical character recognition (OCR)

- The aim in this task is to recognize *handwritten text*.

- Recognition of *isolated characters versus* continuous text.

- *Online* and *offline* systems.

- Today's technology allows to reach high performance in isolated character recognition, close to human skills.

- Commercial systems are available with good performance in *typed isolated characters and acceptable performance for* handwritten characters.
### IMAGES OF HANDWRITTEN CHARACTERS (OCR task)

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</tbody>
</table>
**IMAGE PREPROCESSING:** Basic techniques applicable to OCR

Original digital image (b/w, 128x128) → Minimum inclusion box (b/w, 49x45) → "Aspect ratio" normalization (b/w, 49x49)

Size normalization or scaling (grey 32x32) → Smoothing [optional] (grey, 32x32) → Binarization [optional] (b/w, 32x32)
OCR IMAGE PREPROCESSING (another example)

Original digital image → Minimum inclusion box → Aspect ratio normalization

Original digital image → Size normalization (scaling) → Smoothing (optional) → Binarization (optional)
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FEATURE EXTRACTION in OCR:
Examples of normalized handwritten digits 6 and 7
OCR: Simple example of unidimensional representation

Sixes Histogram

Sevens Histogram
OCR: Simple example of unidimensional representation (cont.)
OCR: Simple example of bidimensional representation

Sixes, Sevens, Eights and Fives in 2 dimensions

Brightness of Upper Part

Brightness of Lower Part

Six
Seven
OCR: Simple example of bidimensional representation (cont.)

Sixes, Sevens and Eights in 2 dimensions

Brightness of Lower Part

Brightness of Upper Part
OCR: Simple example of bidimensional representation (cont.)

Sixes, Sevens, Eights and Fives in 2 dimensions

Brightness of Lower Part

Brightness of Upper Part

Problematic overlapping! A possible solution is to increase dimensionality
A SIMPLE TECHNIQUE for FEATURE EXTRACTION in OCR

“DIRECT GEOMETRIC” METHOD:

- The image goes through a preprocess and a normalization of its (horizontal and vertical) size to some fixed values $I \times J$, which are usually small. This process finally produces a grayscale image (PGM format).

- The brightness of each pixel is considered as a component of a $d$-dimensional vector that represents the image, $d = I \cdot J$.

(255, 255, 235, 137, ... ,120,194, 255, 255)
(8x8 = 64 dimensiones)
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FEATURE EXTRACTION: “LOCAL” METHODS

INTUITIVE KNOWLEDGE

- The *global* representation of an object could change a lot, but *locally* it could exist parts that keep unchanged.

- Both characters are the number “5” but globally the character of the right is more similar to a “0”. Locally this character has more parts of a “5” than “0”.
FEATURE EXTRACTION: “LOCAL” METHODS

- Each image is represented by several smaller parts.

- This parts are selected having some relevant information

- For example some “windows” of the image with some variations of the pixels gray levels.

- Original image of 20x20 pixels. Local representation: 4 local features of 11x11 pixels.
FEATURE EXTRACTION: “LOCAL” METHODS

- Lower dimensionality representations are obtained.
- The relative positions of these local features are not stored.
- The local features are invariant to translations.
ANOTHER EXAMPLE: FACE RECOGNITION

- The local feature approach is interesting when the objects to represent are formed by smaller structural objects.

- Face = eyebrows + eyes + nose + mouth + chin

![Face images]

- Direct representation: A lot of difference between both faces.

- Local representation: Only the relative position of the local features is different.
ANOTHER EXAMPLE: FACE RECOGNITION

- The local features are quite similar:
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STRUCTURAL REPRESENTATION: PRIMITIVE EXTRACTION

- **Geometric/Statistical approach to PR:**
  - Objects are represented as tuples of *features*; i.e., as *points in a vector space*.

- **Structural/Syntactic approach to PR:**
  - Objects are considered to be composed of simpler “subobjects”.
  - The elementary subobjects are called the *primitives*.
  - A simple composition of primitives is *concatenation*.
  - The composition of primitives (or *structure*) is usually represented by *syntactic models* (grammars).
STRUCTURAL REPRESENTATION IN OCR: a basic technique for primitive extraction

*Chain codes of 4 and 8 directions*

**4d:** 0000030330333303333232232222221110010000301001011122223221210101001111222232211001

**8d:** 0000777767666655554544444422110007101123445433110012344543111
CHAIN CODES (cont.)

4 directions

8 directions

00000000000333 ... 22222212101

000000000000333 ... 22222212101
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MICROSCOPIC IMAGES of HUMAN CROMOSOMES

Human male G-bands

1  2  3  4  5  6
7  8  9 10 11 12
13 14 15 16 17 18
19 20 21 22 X  Y
PRIMITIVE EXTRACTION and STRUCTURAL REPRESENTATION of a CROMOSOME

Centromer

Cromosome 2a

"=====CDFDCBBBBBBA===bcdc===DGFB=bccb== ...... ==cffc=CCC==cdb==BCB==dfdcb=====

Primitive Chain"
Examples of cromosomes represented by strings of primitives

1: A====B==aA==a==a==B==a==a==A==a==a==C==d==d==B==a==C==d==C==c==A====b==a
   A==A==a==A==a==A==a==C==d==b==A==a==D==d==b==a==A==a==A==a==a==a==a

2: A==a==A==a==a==A==a==A==a==a==A==a==a==A==a==a==a==A==a==a==a==a==a
   A==A==a==A==a==a==A==a==A==a==a==A==a==a==A==a==a==A==a==a==a==a==a

3: A==B==a==a==a==a==a==a==D==a==C==a==a==B==a==a==a==a==a==a==a==a==a==a
   A==B==a==C==b==A==a==a==a==b==A==a==a==a==a==a==a==a==a==a==a==a==a

4: A====D==b==C==e==C==b==C==b==B==b==B==d==D==D==D==D==D==D==D==D==D==D==D
   A==D==b==a==B==b==A==a==a==a==B==a==a==a==c==C==D==D==D==D==D==D==D==D

5: A==C==a==a==A==a==a==a==a==C==a==a==A==a==a==a==a==a==a==a==a==a==a
   A==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

6: A==B==a==B==b==C==a==a==b==a==b==a==b==a==b==a==b==a==b==a==b==a==b==a
   A==B==a==B==b==C==a==a==b==a==b==a==b==a==b==a==b==a==b==a==b==a==b==a

7: A==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
   A==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

8: A==B==a==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a
   A==B==a==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a

9: A==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
   A==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

10: A==a==B==a==D==b==B==b==C==a==A==d==d==a==a==a==a==a==a==a==a==a==a
    A==A==a==A==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

11: A==B==a==C==a==a==d==d==E==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d
    A==C==a==C==a==C==a==C==a==C==a==C==a==C==a==C==a==C==a==C==a==C==a==C

12: A==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B
    A==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B

13: A==a==B==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==a==B==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

14: A==A==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a
    A==A==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a==B==a

15: A==A==a==E==b==a==a==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==A==a==E==b==a==a==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

16: A==A==E==a==E==e==d==a==b==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==A==E==a==E==e==d==a==b==a==a==a==a==a==a==a==a==a==a==a==a==a==a

17: A==E==a==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==E==a==C==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

18: A==B==a==a==C==a==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d
    A==B==a==a==C==a==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d==d

19: A==a==E==d==d==d==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==a==E==d==d==d==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

20: A==C==a==C==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==C==a==C==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

21: A==a==E==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==a==E==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a

22: A==E==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
    A==E==d==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a==a
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Acquisition of Acoustic Signals

<table>
<thead>
<tr>
<th></th>
<th>Voz telefónica B=3.6khz</th>
<th>Voz Calidad B=8Khz</th>
<th>Audio CD (HI-FI) B=20khz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muestreo (kHz)</td>
<td>8</td>
<td>16</td>
<td>44.1</td>
</tr>
<tr>
<td>Cuantificación (bits)</td>
<td>8 (12)</td>
<td>16 (12)</td>
<td>2x16 (20)</td>
</tr>
<tr>
<td>Flujo de datos (Mbytes/hora)</td>
<td>28.1</td>
<td>112.5</td>
<td>620.2</td>
</tr>
<tr>
<td>Segundos en 1 Mbyte</td>
<td>128</td>
<td>32</td>
<td>5.8</td>
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Nyquist Frequence (violation of the sampling theorem)

Señal original (sin filtrar): F0=600hz; F1=4800hz

Señal muestreada: Fm = 5000 hz < 2\cdot4800 hz

Señal muestreada reconstruida

Señal original (filtrada): F0=600hz; F1=4800hz

Señal muestreada: Fm = 5000 hz > 2\cdot600 hz

Señal muestreada reconstruida
This representation usually leads to confusion in the identification of the objects (utterances of words or sentences).
Preprocessing of Speech Signals

Ventanas de análisis

Señal vocal

/d/ /o/ /s/

Dos espectros instantáneos

Secuencia de espectros

Espectrograma
This representation shows more clearly the discriminative features of the objects, making identification easier.
The Problem of Non-linear Time Variability

The acoustic segments of certain sounds (vowels) can lengthen or shorten arbitrarily, while the duration of other sounds (most of the consonants) varies very little.
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**Primitive Extraction in Voice Signals:**

**Spectra Clustering using Vector Quantification**

The purpose is to represent the sequence of spectra corresponding to the pronunciations of words or sentences by means of *symbol strings*. The process has two phases:

1. **“Codebook” construction**
   - A training set of acoustic vectors is defined
   - This set is partitioned in \( C \) homogeneous groups or *clusters*
   - The mean vector of each *cluster* is taken as a *prototype* or *codeword*, building a *codebook*
   - A tag is assigned to each *codeword*

2. **Vector Quantification**
   - Each spectrum is substituted by the tag of its nearest *codeword*
An Example of Vector Quantification: pronunciations of “ocho”
An Example of Vector Quantification: spectra clustering for the word “ocho”

Vectores acusticos de ochos
agrupados en 4 clases

(projection in 2 dimensions)
An Example of Vector Quantification: spectra clustering for the word “ocho”

Vectores acusticos de ocho
agrupados en 4 clases

The spectra are automatically clustered in “natural classes” associated to the elementary sounds (~phonemes) that appear in the analyzed utterances.
PÁGINA INTENCIONADAMENTE EN BLANCO
Vector Quantification: quadratic error criterium

\[ J(X_1, \ldots, X_C) = \sum_{c} J_c \quad \text{where} \quad J_c = \sum_{\bar{x} \in X_c} \| \bar{x} - \bar{m}_c \|^2 \]

Incremental calculation of \( J \) when moving \( \bar{x} \) from \( X_i \) to \( X_j \)

\[ X_i' = X_i - \{ \bar{x} \} \quad X_j' = X_j + \{ \bar{x} \} \]

\[ \bar{m}'_i = \bar{m}_i - \frac{\bar{x} - \bar{m}_i}{n_i - 1} \quad \bar{m}'_j = \bar{m}_j + \frac{\bar{x} - \bar{m}_j}{n_j + 1} \]

\[ J'_i = J_i - \frac{n_i}{n_i - 1} \| \bar{x} - \bar{m}_i \|^2 \quad J'_j = J_j + \frac{n_j}{n_j + 1} \| \bar{x} - \bar{m}_j \|^2 \]

\[ \Delta J = \frac{n_j}{n_j + 1} \| \bar{x} - \bar{m}_j \|^2 - \frac{n_i}{n_i - 1} \| \bar{x} - \bar{m}_i \|^2 \]
Summary of Preprocessing and Geometric Representation of Speech Signals

Señal vocal (16khz) 8320 muestras

Secuencia de espectros

Espectrograma (21 canales, 100hz) 52x21

Segmentación de Traza (normalización a 10 vectores) 10x21

“Punto” en un Espacio Vectorial de 210 dimensiones
Summary of Preprocessing and Structural Representation of Speech Signals

Señal vocal (16khz) 8320 muestras

Secuencia de espectros

Espectrograma (21 canales, 100hz) 52x21

Cadena de etiquetas acústicas 52 símbolos ($|\Sigma| = 16$)

Cadena reetiquetada fonéticamente