

# Symbolic Model Checking of Infinite-State Systems Using Narrowing

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**Abstract.** Rewriting is a general and expressive way of specifying concurrent systems, where concurrent transitions are axiomatized by rewrite rules. Narrowing is a complete symbolic method for model checking reachability properties. We show that this method can be reinterpreted as a *lifting simulation* relating the original system and the symbolic system associated to the narrowing transitions. Since the narrowing graph can be infinite, this lifting simulation only gives us a semi-decision procedure for the failure of invariants. However, we propose new methods for folding the narrowing tree that can in practice result in finite systems that symbolically simulate the original system and can be used to algorithmically verify its properties. We also show how both narrowing and folding can be used to symbolically model check systems which, in addition, have state predicates, and therefore correspond to Kripke structures on which  $ACTL^*$  and  $LTL$  formulas can be algorithmically verified using such finite symbolic abstractions.

## 1 Introduction

Model checking techniques have proved enormously effective in verification of concurrent systems. However, the standard model checking algorithms only work when the set of states reachable from the given initial state is finite. Various model checking techniques for infinite-state systems exist, but they are less developed than finite-state techniques and tend to place stronger limitations on the kind of systems and/or the properties that can be model checked.

In this work we adopt the rewriting logic point of view, in which a concurrent system can always be axiomatized as a rewrite theory modulo some equational axioms, with system transitions described by rewrite rules. We then propose a new narrowing-based method for model checking such, possibly infinite-state, systems under reasonable assumptions. The key insight is that the well-known theorem on the completeness of narrowing (which for rewrite theories whose rules need not be convergent have to satisfy a topmost restriction) can be reinterpreted as a *lifting simulation* between two systems, namely, between the initial model associated to the rewrite theory (which describes our system of interest), and a “symbolic abstraction” of such a system by the narrowing relation.

The narrowing relation itself may still lead to an infinite-state system. Even then, narrowing already gives us a semi-decision procedure for finding failures of

invariants. To obtain a finite-state abstraction, we then define a second simulation by *folding* the narrowing-based abstraction, using a generalization criterion to fold the possibly infinite narrowing tree into a finite graph. There is no guarantee that such a folding will always be finite. But we think that such foldings can be finite in many practical cases and give several examples of finite concurrent system abstractions of infinite systems that can be obtained in this way and can be used to verify properties of infinite systems.

Our work applies not only to the model checking of invariants, but also to the model checking of  $ACTL^*$  and  $LTL$  temporal logic formulas; not just for one initial state, but for a possibly infinite, symbolically described set of initial states. We therefore also provide results about the  $ACTL^*$  and  $LTL$  model checking of concurrent systems axiomatized as rewrite theories. For such temporal logic model checking we have to perform narrowing in two different dimensions: (i) in the dimension of *transitions*, as already explained above; and (ii) in the dimensions of *state predicates*, because they are not defined in general for arbitrary terms with variables, but only for suitable substitution instances. Again, our narrowing techniques, when successful in folding the system into a finite-state abstraction, allow the use of standard model checking algorithms to verify  $ACTL^*$  and  $LTL$  properties of the corresponding infinite-state systems.

After some preliminaries in Section 2, we consider narrowing for model checking invariants of transition systems in Section 3, and narrowing for model checking temporal logic formulas on Kripke structures in Section 4. We conclude in Section 5. Throughout we use Lamport’s infinite-state “bakery” protocol as the source of various examples. Other examples based on a readers-writers protocol and the proofs of all technical results are included in [16].

## 1.1 Related work

The idea that narrowing in its reachability sense should be used as a method for analyzing concurrent systems and should fit within a wider spectrum of analysis capabilities, was suggested in [26,13], and was fully developed in [24]. The application of this idea to the verification of cryptographic protocols has been further developed by the authors in collaboration with Catherine Meadows and has been used as the basis of the Maude-NPA protocol analyzer [15]. In relation to such previous work, we contribute several new ideas, including the use of lifting simulations, the folding of the narrowing graph by a generalization criterion, and the new techniques for the verification of  $ACTL^*$  and  $LTL$  properties.

The methods proposed in this paper are complementary to other infinite-state model checking methods, of which narrowing is one. What narrowing has in common with various infinite-state model checking analyses is the idea of representing sets of states *symbolically*, and to perform reachability analysis to verify properties. The symbolic representations vary from approach to approach. String and multiset grammars are often used to symbolically compute reachability sets, sometimes in conjunction with descriptions of the systems as rewrite theories [5,4], and sometimes in conjunction with learning algorithms [33]. Tree automata are also used for symbolic representation [19,30]. In general,

like narrowing, some of these methods are only semi-decision procedures; but by restricting the classes of systems and/or the properties being analyzed, and by sometimes using acceleration or learning techniques, actual algorithms can be obtained for suitable subclasses: see the references above and also [6,7,14,18].

Two infinite-state model checking approaches closer in spirit to ours are: (i) the “constraint-based multiset rewriting” of Delzanno [12,11], where the infinity of a concurrent system is represented by the use of constraints (over integer or real numbers) and reachability analysis is performed by rewriting with a constraint store to which more constraints are added and checked for satisfiability or failure; and (ii) the logic-programming approach of [3], where simulations/bisimulations of labeled transition systems and symbolic representations of them using terms with variables and logic programming are studied. In spite of their similarities, the technical approaches taken in (i) and (ii) are quite different from ours. In (i), the analogue of narrowing is checking satisfiability of the constraint store; whereas in (ii) the main focus is on analyzing process calculi and on developing effective techniques using tabled logic programming to detect when a simulation or bisimulation exists.

Our work is also related to abstraction techniques, e.g., [8,22,20,21,31], which can sometimes collapse an infinite-state system into a finite-state one. In particular, it is related to, and complements, abstraction techniques for rewrite theories such as [29,23,17]. In fact, all the simulations we propose, especially the ones involving folding, can be viewed as suitable abstractions. From this point of view, our results provide new methods for automatically defining correct abstractions in a symbolic way. There is, finally, related work on computing finite representations of the search space associated by narrowing to an expression in a rewrite theory, e.g., for computing regular expressions denoting a possibly infinite set of unifiers in [2], or for partial evaluation in [1]. However, these works have a different motivation and do not consider applications to simulation/bisimulation issues, although they contain notions of correctness and completeness suitable for such applications.

## 2 Preliminaries

We follow the classical notation and terminology from [32] for term rewriting and from [25,27] for rewriting logic and order-sorted notions. We assume an *order-sorted signature*  $\Sigma$  with a finite poset of sorts  $(S, \leq)$  and a finite number of function symbols. We furthermore assume that: (i) each connected component in the poset ordering has a top sort, and for each  $s \in S$  we denote by  $[s]$  the top sort in the component of  $s$ ; and (ii) for each operator declaration  $f : s_1 \times \dots \times s_n \rightarrow s$  in  $\Sigma$ , there is also a declaration  $f : [s_1] \times \dots \times [s_n] \rightarrow [s]$ . We assume an  $S$ -sorted family  $\mathcal{X} = \{\mathcal{X}_s\}_{s \in S}$  of disjoint variable sets with each  $\mathcal{X}_s$  countably infinite.  $\mathcal{T}_\Sigma(\mathcal{X})_s$  is the set of terms of sort  $s$ , and  $\mathcal{T}_{\Sigma,s}$  is the set of ground terms of sort  $s$ . We write  $\mathcal{T}_\Sigma(\mathcal{X})$  and  $\mathcal{T}_\Sigma$  for the corresponding term algebras. The set of positions of a term  $t$  is written  $Pos(t)$ , and the set of non-variable positions  $Pos_\Sigma(t)$ . The root of a term is  $\lambda$ . The subterm of  $t$  at position  $p$  is  $t|_p$  and  $t[u]_p$

is the subterm  $t|_p$  in  $t$  replaced by  $u$ . A *substitution*  $\sigma$  is a sorted mapping from a finite subset of  $\mathcal{X}$ , written  $Dom(\sigma)$ , to  $\mathcal{T}_\Sigma(\mathcal{X})$ . The set of variables introduced by  $\sigma$  is  $Ran(\sigma)$ . The identity substitution is *id*. Substitutions are homomorphically extended to  $\mathcal{T}_\Sigma(\mathcal{X})$ . The restriction of  $\sigma$  to a set of variables  $V$  is  $\sigma|_V$ .

A  $\Sigma$ -*equation* is an unoriented pair  $t = t'$ , where  $t, t' \in \mathcal{T}_\Sigma(\mathcal{X})_s$  for some sort  $s \in \mathbf{S}$ . Given  $\Sigma$  and a set  $E$  of  $\Sigma$ -equations such that  $\mathcal{T}_{\Sigma,s} \neq \emptyset$  for every sort  $s$ , order-sorted equational logic induces a congruence relation  $=_E$  on terms  $t, t' \in \mathcal{T}_\Sigma(\mathcal{X})$  (see [27]). Throughout this paper we assume that  $\mathcal{T}_{\Sigma,s} \neq \emptyset$  for every sort  $s$ . The *E-subsumption* order on terms  $\mathcal{T}_\Sigma(\mathcal{X})_s$ , written  $t \preceq_E t'$  (meaning that  $t'$  is more general than  $t$ ), holds if  $\exists \sigma : t =_E \sigma(t')$ . The *E-renaming* equivalence on terms  $\mathcal{T}_\Sigma(\mathcal{X})_s$ , written  $t \approx_E t'$ , holds if  $t \preceq_E t'$  and  $t' \preceq_E t$ . We extend  $=_E$ ,  $\approx_E$ , and  $\preceq_E$  to substitutions in the expected way. An *E-unifier* for a  $\Sigma$ -equation  $t = t'$  is a substitution  $\sigma$  s.t.  $\sigma(t) =_E \sigma(t')$ . A *complete* set of *E-unifiers* of an equation  $t = t'$  is written  $CSU_E(t = t')$ . We say  $CSU_E(t = t')$  is *finitary* if it contains a finite number of *E-unifiers*. This notion can be extended to several equations, written  $CSU_E(t_1 = t'_1 \wedge \dots \wedge t_n = t'_n)$ .

A *rewrite rule* is an oriented pair  $l \rightarrow r$ , where  $l \notin \mathcal{X}$  and  $l, r \in \mathcal{T}_\Sigma(\mathcal{X})_s$  for some sort  $s \in \mathbf{S}$ . An (*unconditional*) *order-sorted rewrite theory* is a triple  $\mathcal{R} = (\Sigma, E, R)$  with  $\Sigma$  an order-sorted signature,  $E$  a set of  $\Sigma$ -equations, and  $R$  a set of rewrite rules. A *topmost rewrite theory* is a rewrite theory s.t. for each  $l \rightarrow r \in R$ ,  $l, r \in \mathcal{T}_\Sigma(\mathcal{X})_{\text{State}}$  for a top sort **State**,  $r \notin \mathcal{X}$ , and no operator in  $\Sigma$  has **State** as an argument sort. The rewriting relation  $\rightarrow_R$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  is  $t \xrightarrow{p}_R t'$  (or  $\rightarrow_R$ ) if  $p \in Pos_\Sigma(t)$ ,  $l \rightarrow r \in R$ ,  $t|_p = \sigma(l)$ , and  $t' = t[\sigma(r)]_p$  for some  $\sigma$ . The relation  $\rightarrow_{R/E}$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  is  $=_E; \rightarrow_R; =_E$ . Note that  $\rightarrow_{R/E}$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  induces a relation  $\rightarrow_{R/E}$  on  $\mathcal{T}_{\Sigma/E}(\mathcal{X})$  by  $[t]_E \rightarrow_{R/E} [t']_E$  iff  $t \rightarrow_{R/E} t'$ . When  $\mathcal{R} = (\Sigma, E, R)$  is a topmost rewrite theory we can safely restrict ourselves to the rewriting relation  $\rightarrow_{R,E}$  on  $\mathcal{T}_\Sigma(\mathcal{X})$ , where  $t \xrightarrow{\Delta}_{R,E} t'$  (or  $\rightarrow_{R,E}$ ) if  $l \rightarrow r \in R$ ,  $t =_E \sigma(l)$ , and  $t' = \sigma(r)$ . Note that  $\rightarrow_{R,E}$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  induces a relation  $\rightarrow_{R,E}$  on  $\mathcal{T}_{\Sigma/E}(\mathcal{X})$  by  $[t]_E \rightarrow_{R,E} [t']_E$  iff  $\exists w \in \mathcal{T}_\Sigma(\mathcal{X})$  s.t.  $t \rightarrow_{R,E} w$  and  $w =_E t'$ . The narrowing relation  $\rightsquigarrow_R$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  is  $t \xrightarrow{p,\sigma}_R t'$  (or  $\overset{\sigma}{\rightsquigarrow}_R, \rightsquigarrow_R$ ) if  $p \in Pos_\Sigma(t)$ ,  $l \rightarrow r \in R$ ,  $\sigma \in CSU_\emptyset(t|_p = l)$ , and  $t' = \sigma(t[r]_p)$ . Assuming that  $E$  has a finitary and complete unification algorithm, the narrowing relation  $\rightsquigarrow_{R,E}$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  is  $t \xrightarrow{p,\sigma}_{R,E} t'$  (or  $\overset{\sigma}{\rightsquigarrow}_{R,E}, \rightsquigarrow_{R,E}$ ) if  $p \in Pos_\Sigma(t)$ ,  $l \rightarrow r \in R$ ,  $\sigma \in CSU_E(t|_p = l)$ , and  $t' = \sigma(t[r]_p)$ . Note that  $\rightsquigarrow_{R,E}$  on  $\mathcal{T}_\Sigma(\mathcal{X})$  induces a relation  $\rightsquigarrow_{R,E}$  on  $\mathcal{T}_{\Sigma/E}(\mathcal{X})$  by  $[t]_E \overset{\sigma}{\rightsquigarrow}_{R,E} [t']_E$  iff  $\exists w \in \mathcal{T}_\Sigma(\mathcal{X}) : t \overset{\sigma}{\rightsquigarrow}_{R,E} w$  and  $w =_E t'$ . Note that, since we will only consider topmost rewrite theories, we avoid any coherence problems, and, as pointed above for  $\rightarrow_{R/E}$  and  $\rightarrow_{R,E}$ , the narrowing relation  $\rightsquigarrow_{R,E}$  achieves the same effect as a more general narrowing relation  $\rightsquigarrow_{R/E}$  (see [24]).

### 3 Narrowing-based Reachability Analysis

A rewrite theory  $\mathcal{R} = (\Sigma, E, R)$  specifies a transition system  $\mathcal{T}_\mathcal{R}$  whose states are elements of the initial algebra  $\mathcal{T}_{\Sigma/E}$ , and whose transitions are specified by

R. Before discussing the narrowing-based reachability analysis of the system  $\mathcal{T}_{\mathcal{R}}$ , we review some basic notions about transition systems.

**Definition 1 (Transition System).** A transition system is written  $\mathcal{A} = (A, \rightarrow)$ , where  $A$  is a set of states, and  $\rightarrow$  is a transition relation between states, i.e.,  $\rightarrow \subseteq A \times A$ . We write  $\mathcal{A} = (A, \rightarrow, I)$  when  $I \subseteq A$  is a set of initial states.

Frequently, we will restrict our attention to a set of initial states in the transition system and, therefore, to the subsystem of states and transitions reachable from those initial states. However, we can obtain a useful approximation of such a reachable subsystem by using a *folding relation* in order to shrink the associated transition system, i.e., to collapse several states into a previously seen state according to some criteria.

**Definition 2 (Folding Reachable Transition Subsystem).** Given  $\mathcal{A} = (A, \rightarrow, I)$  and a relation  $G \subseteq A \times A$ , the reachable subsystem from  $I$  in  $A$  with folding  $G$  is written  $\mathcal{R}eac\mathcal{H}_{\mathcal{A}}^G(I) = (\mathcal{R}eac\mathcal{H}_{\mathcal{A}}^G(I), \rightarrow^G, I)$ , where

$$\begin{aligned} \mathcal{R}eac\mathcal{H}_{\mathcal{A}}^G(I) &= \bigcup_{n \in \mathbb{N}} \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_n, \\ \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_0 &= I, \\ \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_{n+1} &= \{y \in A \mid (\exists z \in \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_n : z \rightarrow y) \wedge \\ &\quad (\nexists k \leq n, w \in \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_k : y G w)\}, \\ \rightarrow^G &= \bigcup_{n \in \mathbb{N}} \rightarrow_{n+1}^G, \\ x \rightarrow_{n+1}^G y &\begin{cases} \text{if } x \in \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_n, y \in \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_{n+1}, x \rightarrow y; \text{ or} \\ \text{if } x \in \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_n, y \notin \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_{n+1}, \\ \quad \exists k \leq n : y \in \mathcal{F}r\mathcal{O}n\mathcal{T}i\mathcal{E}r_{\mathcal{A}}^G(I)_k, \exists w : (x \rightarrow w \wedge w G y) \end{cases} \end{aligned}$$

Note that, the more general the relation  $G$ , the greater the chances of  $\mathcal{R}eac\mathcal{H}_{\mathcal{A}}^G(I)$  being a finite transition system. In this paper, we consider only folding relations  $G \in \{=, \approx, \preceq\}$  on transition systems whose state set is  $\mathcal{T}_{\Sigma/E}(\mathcal{X})_s$  for a given sort  $s$ . We plan to study other folding relations. For  $=_A = \{(a, a) \mid a \in A\}$ , we write  $\mathcal{R}eac\mathcal{H}_{\mathcal{A}}(I)$  for the transition system  $\mathcal{R}eac\mathcal{H}_{\mathcal{A}}^{=_A}(I)$ , which is the standard notion of reachable subsystem. We are furthermore interested in comparisons between different transition systems, for which we use the notions of simulation, lifting simulation, and bisimulation.

**Definition 3 (Simulation, lifting simulation, and bisimulation).** Let  $\mathcal{A} = (A, \rightarrow_{\mathcal{A}})$  and  $\mathcal{B} = (B, \rightarrow_{\mathcal{B}})$  be two transition systems. A simulation from  $\mathcal{A}$  to  $\mathcal{B}$ , written  $\mathcal{A} H \mathcal{B}$ , is a relation  $H \subseteq A \times B$  such that  $a H b$  and  $a \rightarrow_{\mathcal{A}} a'$  implies that there exists  $b' \in B$  such that  $a' H b'$  and  $b \rightarrow_{\mathcal{B}} b'$ . Given  $\mathcal{A} = (A, \rightarrow_{\mathcal{A}}, I_{\mathcal{A}})$  and  $\mathcal{B} = (B, \rightarrow_{\mathcal{B}}, I_{\mathcal{B}})$ ,  $H$  is a simulation from  $\mathcal{A}$  to  $\mathcal{B}$  if  $(A, \rightarrow_{\mathcal{A}}) H (B, \rightarrow_{\mathcal{B}})$  and  $\forall a \in I_{\mathcal{A}}, \exists b \in I_{\mathcal{B}}$  s.t.  $a H b$ . A simulation  $H$  from  $(A, \rightarrow_{\mathcal{A}})$  to  $(B, \rightarrow_{\mathcal{B}})$  (resp. from  $(A, \rightarrow_{\mathcal{A}}, I_{\mathcal{A}})$  to  $(B, \rightarrow_{\mathcal{B}}, I_{\mathcal{B}})$ ) is a bisimulation if  $H^{-1}$  is a simulation from  $(B, \rightarrow_{\mathcal{B}})$  to  $(A, \rightarrow_{\mathcal{A}})$  (resp. from  $(B, \rightarrow_{\mathcal{B}}, I_{\mathcal{B}})$  to  $(A, \rightarrow_{\mathcal{A}}, I_{\mathcal{A}})$ ). We call a simulation  $(A, \rightarrow_{\mathcal{A}}, I_{\mathcal{A}}) H (B, \rightarrow_{\mathcal{B}}, I_{\mathcal{B}})$  a lifting simulation if for each finite sequence  $b_0 \rightarrow_{\mathcal{B}} b_1 \rightarrow_{\mathcal{B}} b_2 \rightarrow_{\mathcal{B}} \dots \rightarrow_{\mathcal{B}} b_n$  with  $b_0 \in I_{\mathcal{B}}$ , there exists a finite sequence  $a_0 \rightarrow_{\mathcal{A}} a_1 \rightarrow_{\mathcal{A}} a_2 \rightarrow_{\mathcal{A}} \dots \rightarrow_{\mathcal{A}} a_n$  with  $a_0 \in I_{\mathcal{A}}$  such that  $a_i H b_i$  for  $0 \leq i \leq n$ .

Note that a lifting simulation is not necessarily a bisimulation. A lifting simulation is a simulation which ensures that false finite counterexamples do not exist. It is easy to see that simulations, lifting simulations, and bisimulations compose, that is, if  $\mathcal{A} \ H \ \mathcal{B} \ K \ \mathcal{C}$  are simulations (resp. lifting simulations, resp. bisimulations), then  $\mathcal{A} \ H;K \ \mathcal{C}$  is a simulation (resp. lifting simulation, resp. bisimulation). In fact, we have associated categories, with transition systems as objects and simulations (resp. lifting simulations, resp. bisimulations) as morphisms.

In rewriting logic we usually specify a concurrent system as a topmost<sup>3</sup> rewrite theory  $\mathcal{R} = (\Sigma, E, R)$ , where states are  $E$ -equivalence classes of ground terms of a concrete top sort **State**, i.e., elements in  $\mathcal{T}_{\Sigma/E, \text{State}}$ , and transitions are rewrite rules  $l \rightarrow r$  for  $l, r \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\text{State}}$  that rewrite states into states. We can describe the operational behavior of the concurrent system by an associated transition system.

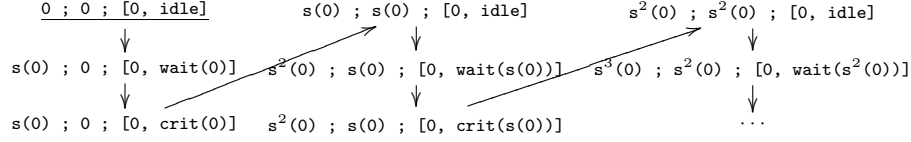
**Definition 4 ( $\mathcal{T}_{\mathcal{R}}$ -Transition System).** *Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort **State**. We define the transition system  $\mathcal{T}_{\mathcal{R}} = (\mathcal{T}_{\Sigma/E, \text{State}}, \rightarrow_{R, E})$ .*

*Example 1.* Consider a simplified version of Lamport’s bakery protocol, in which we have several processes, each denoted by a natural number, that achieve mutual exclusion between them by the usual method common in bakeries and deli shops: there is a number dispenser, and customers are served in sequential order according to the number that they hold. This system can be specified as an order-sorted topmost rewrite theory in Maude<sup>4</sup> as follows:

```
fmod BAKERY-SYNTAX is
  sort Nat .
  op 0 : -> Nat .
  op s : Nat -> Nat .
```

<sup>3</sup> Obviously, not all concurrent systems need to have a topmost rewrite theory specification. However, as explained in [24], many concurrent systems of interest, including the vast majority of distributed algorithms, admit topmost specifications. For example, concurrent object-oriented systems whose state is a multiset of objects and messages can be given a topmost specification by enclosing the system state in a top operator. Even hierarchical distributed systems of the “Russian doll” kind can likewise be so specified, provided that the boundaries defining such hierarchies are not changed by transitions.

<sup>4</sup> The Maude syntax is so close to the corresponding mathematical notation for defining rewrite theories as to be almost self-explanatory. The general point to keep in mind is that each item: a sort, a subsort, an operation, an equation, a rule, etc., is declared with an obvious keyword: **sort**, **subsort**, **op**, **eq**, **rl**, etc., with each declaration ended by a space and a period. A rewrite theory  $\mathcal{R} = (\Sigma, E, R)$  is defined with the signature  $\Sigma$  using keyword **op**, equations in  $E$  are specified using keyword **eq** or keywords **assoc**, **comm** and **id**: (for associativity, commutativity, and identity, respectively) appearing in an operator declaration, and rules in  $R$  using keyword **rl**. Another important point is the use of “mix-fix” user-definable syntax, with the argument positions specified by underbars; for example: **if.then.else.fi**. We write the sort of a variable using keyword **var** or after its name and a colon, e.g. **X:Nat**.



**Fig. 1.** Infinite transition system  $\mathcal{R}eac\hbar_{\mathcal{T}\mathcal{R}}(0 ; 0 ; [0, \text{idle}])$

```

sorts ModeIdle ModeWait ModeCrit Mode .
subsorts ModeIdle ModeWait ModeCrit < Mode .
sorts ProcIdle ProcWait Proc ProcIdleSet ProcWaitSet ProcSet .
subsorts ProcIdle < ProcIdleSet .
subsorts ProcWait < ProcWaitSet .
subsorts ProcIdle ProcWait < Proc < ProcSet .
subsorts ProcIdleSet < ProcWaitSet < ProcSet .
op idle : -> ModeIdle .
op wait : Nat -> ModeWait .
op crit : Nat -> ModeCrit .
op [_,_] : Nat ModeIdle -> ProcIdle .
op [_,_] : Nat ModeWait -> ProcWait .
op [_,_] : Nat Mode -> Proc .
op none : -> ProcIdleSet .
op __ : ProcIdleSet ProcIdleSet -> ProcIdleSet [assoc comm id: none] .
op __ : ProcWaitSet ProcWaitSet -> ProcWaitSet [assoc comm id: none] .
op __ : ProcSet ProcSet -> ProcSet [assoc comm id: none] .
sort State .
op _;-_- : Nat Nat ProcSet -> State .
endfm
mod BAKERY is
  protecting BAKERY-SYNTAX .
  var PS : ProcSet .
  vars N M K : Nat .
  rl N ; M ; [K, idle] PS => s(N) ; M ; [K, wait(N)] PS .
  rl N ; M ; [K, wait(M)] PS => N ; M ; [K, crit(M)] PS .
  rl N ; M ; [K, crit(M)] PS => N ; s(M) ; [K, idle] PS .
endm

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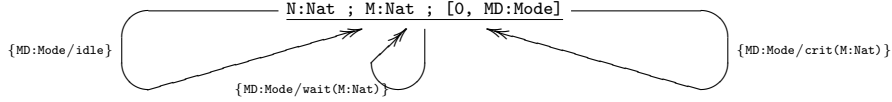
Given the initial state  $t_1 = "0 ; 0 ; [0, \text{idle}]"$ , where the first natural is the last distributed ticket and the second one is the value of the current ticket number accepted in critical section, the infinite transition system  $\mathcal{R}eac\hbar_{\mathcal{T}\mathcal{R}}(t_1)$  is depicted in Figure 1. We will graphically identify initial states by underlining them.

Narrowing calculates the most general rewriting sequences associated to a term. We can exploit this generality and use narrowing as a lifting simulation of rewriting. We write  $\mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^\circ$  for the set of  $E$ -equivalence classes of terms of sort  $\text{State}$  excluding variables, i.e.,  $\mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^\circ = \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}} \setminus \mathcal{X}_{\text{State}}$ . We can define the transition system associated to narrowing as follows.

**Definition 5 ( $\mathcal{N}_{\mathcal{R}}$ -Transition System).** *Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . We define a transition system  $\mathcal{N}_{\mathcal{R}} = (\mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^\circ, \rightsquigarrow_{R,E})$ .*

Note that we exclude variables in Definition 5, since the relation  $\rightsquigarrow_{R,E}$  is not defined on them.

Theorem 1 below relates the transition systems associated to narrowing and rewriting. Note that we do not have a bisimulation in general, since a term



**Fig. 2.** Finite transition system  $Reach_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(N: \text{Nat} ; M: \text{Nat} ; [0, \text{MD: Mode}])$

$t \in \mathcal{T}_{\Sigma}(\mathcal{X})$  may have narrowing steps with incomparable substitutions  $\sigma_1, \dots, \sigma_k$ , i.e., given  $i \neq j$ ,  $\sigma_i(t)$  may disable the rewriting step performed on  $\sigma_j(t)$  and viceversa. Our results are based on the following result from [24].

**Lemma 1 (Topmost Completeness).** [24] *For  $\mathcal{R} = (\Sigma, E, R)$  a topmost theory, let  $t \in \mathcal{T}_{\Sigma}(\mathcal{X})$  be a term that is not a variable, and let  $V$  be a set of variables containing  $\text{Var}(t)$ . For some substitution  $\rho$ , let  $\rho(t) \rightarrow_{R/E} t'$  using the rule  $l \rightarrow r$  in  $R$ . Then there are  $\sigma, \theta, t''$  such that  $t \overset{\sigma}{\rightsquigarrow}_{R,E} t''$  using the same rule  $l \rightarrow r$ ,  $t''$  is not a variable,  $\rho|_V =_E (\sigma \circ \theta)|_V$ , and  $\theta(t'') =_E t'$ .*

Given a subset  $U \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_s$ , we define the set of *ground instances* of  $U$  as  $\llbracket U \rrbracket = \{[t]_E \in \mathcal{T}_{\Sigma/E,s} \mid \exists [t']_E \in U \text{ s.t. } t \preceq_E t'\}$ . Note that  $U$  may be a finite set, whereas  $\llbracket U \rrbracket$  can often be an infinite set. This gives us a symbolic way of describing possibly infinite sets of initial states in  $\mathcal{T}_{\mathcal{R}}$ , which will be very useful for model checking purposes.

**Theorem 1 (Lifting simulation by narrowing).** *Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . Let  $U \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$ . The relation  $\preceq_E$  defines two lifting simulations:  $\mathcal{T}_{\mathcal{R}} \preceq_E \mathcal{N}_{\mathcal{R}}$  and  $Reach_{\mathcal{T}_{\mathcal{R}}}(\llbracket U \rrbracket) \preceq_E Reach_{\mathcal{N}_{\mathcal{R}}}(U)$ .*

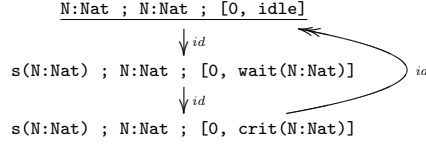
Since  $\mathcal{N}_{\mathcal{R}}$  is typically infinite, for a set  $U \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$  of initial states and a relation  $G \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ} \times \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$ , to obtain a finite abstraction we may be interested in the reachable subsystem from  $U$  in  $\mathcal{N}_{\mathcal{R}}$  with folding  $G$ , i.e., in the transition system  $Reach_{\mathcal{N}_{\mathcal{R}}}^G(U)$ .

*Example 2.* Consider Example 1 and let  $t_2 = \text{“}N: \text{Nat} ; M: \text{Nat} ; [0, \text{MD: Mode}] \text{”}$ . The finite transition system  $Reach_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t_2)$  is depicted in Figure 2. In the case of narrowing, we will graphically tie the substitution computed by each narrowing step to the proper transition arrow. Also, when a transition step is making use of the folding relation  $G$ , i.e., when it is not a normal rewriting/narrowing step but a combination of rewriting/narrowing and folding with the relation  $G$ , we mark the arrow with a double arrowhead.

Since a transition system usually includes a set of initial states, we can extend Theorem 1 to a folding relation  $G$ , to obtain a more specific (and in some sense more powerful) result. For this we need the following compatibility requirement for a folding relation  $G$ .

**Definition 6 ( $\rightsquigarrow_{R,E}$ -equivalent relation).** *Let  $\mathcal{R} = (\Sigma, E, R)$  be a rewrite theory. The binary relation  $G \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X}) \times \mathcal{T}_{\Sigma/E}(\mathcal{X})$  is called  $\rightsquigarrow_{R,E}$ -equivalent*





**Fig. 3.** Finite transition system  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(\text{N:Nat ; N:Nat ; [0, idle]})$

if for  $[t]_E, [t']_E, [w]_E \in \mathcal{T}_{\Sigma/E}(\mathcal{X})$  such that  $t G w$  and  $t \rightsquigarrow_{R,E} t'$  using rule  $l \rightarrow r$ , there is  $[w']_E \in \mathcal{T}_{\Sigma/E}(\mathcal{X})$  such that  $w \rightsquigarrow_{R,E} w'$  using rule  $l \rightarrow r$  and  $t' G w'$ .

**Lemma 2** ( $\rightsquigarrow_{R,E}$ -equivalence of  $G$ ). Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . The relations  $\{=_E, \approx_E, \leq_E\}$  on  $\mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$  are  $\rightsquigarrow_{R,E}$ -equivalent.

**Theorem 2** (Simulation by  $G$ -narrowing). Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . Let  $U \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$  and  $G \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ} \times \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$  be  $\rightsquigarrow_{R,E}$ -equivalent. The relation  $G$  then defines a simulation  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}(U) G \text{Reach}_{\mathcal{N}_{\mathcal{R}}}^G(U)$ .

We can obtain a bisimulation when every narrowing step of a transition system computes the identity substitution. Intuitively, every possible (ground) rewriting sequence is represented in its most general way, since narrowing does not further instantiate states in the narrowing tree. The following results rephrase Theorem 1, Lemma 2, and Theorem 2 above for bisimulations.

**Theorem 3** (Bisimulation by narrowing). Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . Let  $U \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$ . Let each transition in  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}(U)$  be of the form  $[t]_E \rightsquigarrow_{R,E}^{\text{id}} [t']_E$ . The relation  $\leq_E$  then defines a bisimulation  $\text{Reach}_{\mathcal{T}_{\mathcal{R}}}(\llbracket U \rrbracket) \leq_E \text{Reach}_{\mathcal{N}_{\mathcal{R}}}(U)$ .

**Lemma 3** ( $\rightsquigarrow_{R,E}$ -equivalence of  $G^{-1}$ ). Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . Let  $T \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$  be such that for each  $[t]_E, [t']_E \in T$ ,  $[t]_E \rightsquigarrow_{R,E}^{\sigma} [t']_E$  implies  $\sigma = \text{id}$ . The relations  $\{=_{E^{-1}}, \approx_{E^{-1}}, \leq_{E^{-1}}\}$  on  $T$  are  $\rightsquigarrow_{R,E}$ -equivalent.

**Theorem 4** (Bisimulation by  $G$ -narrowing). Let  $\mathcal{R} = (\Sigma, E, R)$  be a topmost rewrite theory with a top sort  $\text{State}$ . Let  $G \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ} \times \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$  and  $G^{-1}$  be  $\rightsquigarrow_{R,E}$ -equivalent. Let  $U \subseteq \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^{\circ}$ . Let each transition in  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^G(U)$  be of the form  $[t]_E \rightsquigarrow_{R,E}^{\text{id}} [t']_E$ . The relation  $G$  then defines a bisimulation  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}(U) G \text{Reach}_{\mathcal{N}_{\mathcal{R}}}^G(U)$ .

*Example 3.* Consider Example 1 and  $t_3 = \text{“N:Nat ; N:Nat ; [0, idle]”}$ . The finite transition system  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t_3)$  is depicted in Figure 3. Note that every transition has the  $\text{id}$  substitution. Therefore, by Theorems 1 and 4, we have a bisimulation between the infinite transition system  $\text{Reach}_{\mathcal{T}_{\mathcal{R}}}(0 ; 0 ; [0, \text{idle}])$  shown in Figure 1 and  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(\text{N:Nat ; N:Nat ; [0, idle]})$  in Figure 3.

Note that the narrowing-based methods we have presented allow us to answer *reachability questions* of the form  $(\exists \vec{x}) t \rightarrow^* t'$ . That is, given a set of initial states  $\llbracket t \rrbracket$  we want to know whether from some state in  $\llbracket t \rrbracket$  we can reach a state in  $\llbracket t' \rrbracket$ . The fact that narrowing provides a *lifting* simulation of the system  $\mathcal{T}_{\mathcal{R}}$  means that it is a *complete* semi-decision procedure for answering such reachability questions: the above existential formula holds in  $\mathcal{T}_{\mathcal{R}}$  if and only if from  $t$  we can reach by narrowing a term that  $E$ -unifies with  $t'$ . In particular, narrowing is very useful for verification of *invariants*. Let  $p \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\text{State}}$  be a pattern representing the set-theoretic complement of an invariant. Then, the reachability formula  $\nexists \vec{x} : t \rightarrow^* p$  corresponds to the satisfaction of the invariant for the set of initial states  $\llbracket t \rrbracket$ . Therefore, narrowing provides a semi-decision procedure for the *violation* of invariants. Furthermore, the invariant holds iff  $p$  does not  $E$ -unify with any term in  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}(t)$ . It also holds if  $p$  does not  $E$ -unify with any term in  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t)$ , which is a decidable question if  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t)$  is finite. If  $p$  does  $E$ -unify with some term in  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t)$ , in general the invariant may or may not hold: we need to check whether this corresponds to a real narrowing sequence.

*Example 4.* Consider Example 1 and the following initial state with two processes  $t_4 = \text{“N:Nat ; N:Nat ; [0, idle] [s(0), idle]”}$ . The finite transition system  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t_4)$  is depicted in Figure 4. Note that we have a bisimulation between  $\text{Reach}_{\mathcal{T}_{\mathcal{R}}}(\llbracket t_4 \rrbracket)$  and  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^{\leq E}(t_4)$ . Consider the following pattern identifying that the critical section property has been violated

“N:Nat ; M:Nat ; [0, crit(C1:Nat)] [s(0), crit(C2:Nat)]”.

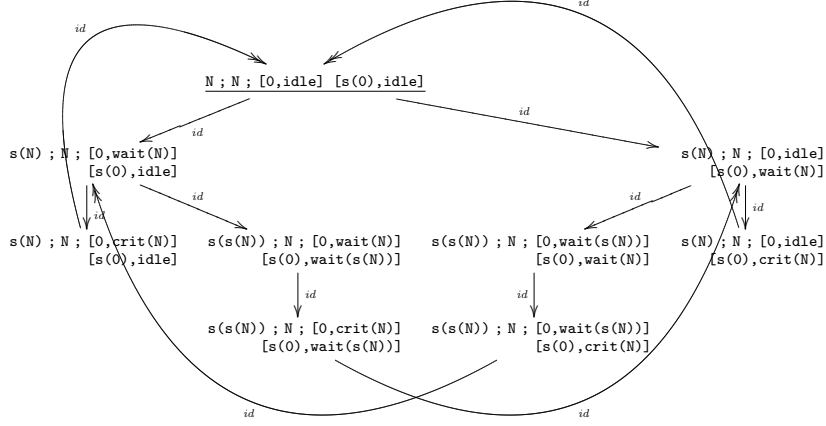
We can check that the pattern does not unify with any state in the transition system of Figure 4, and thus this bad pattern is unreachable from any initial state being an instance of  $t_4$ . This provides a verification of the *mutual exclusion* property for the infinite-state BAKERY protocol, not just from a single initial state, but from an infinite set  $\llbracket t_4 \rrbracket$  of initial states.

Note, finally, that, for  $U$  a set of of initial states, even if the transition system  $\text{Reach}_{\mathcal{T}_{\mathcal{R}}}(\llbracket U \rrbracket)$  is finite, the transition system  $\text{Reach}_{\mathcal{N}_{\mathcal{R}}}^G(U)$  can be much smaller. Furthermore, the set  $U$  is typically finite, whereas the set  $\llbracket U \rrbracket$  is typically infinite, making it impossible to model check an invariant from each initial state by finitary methods. In all these ways, narrowing allows algorithmic verification of invariants in many infinite-state systems, and also in finite-state systems whose size may make them unfeasible to use standard model checking techniques.

## 4 Narrowing-based ACTL\* Model Checking

Due to space restrictions, we omit many technical details and results of this section, which can be found in [16].

Model checking [9] is the most successful verification technique for temporal logics. When we perform model checking, we use *Kripke structures* [9] to represent the state search space, which are just transition systems to which we have

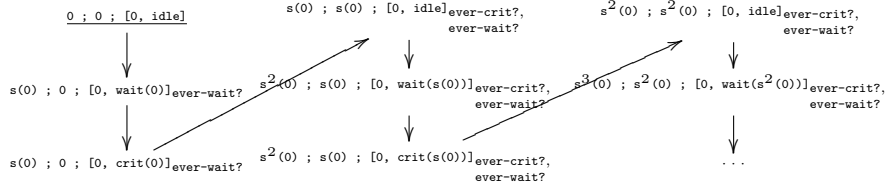


**Fig. 4.** Finite transition system  $\mathcal{R}eac\mathcal{H}_{\mathcal{N}\mathcal{R}}^{\mathcal{E}}(\mathbb{N}:\mathbb{N}at ; \mathbb{N}:\mathbb{N}at ; [0, idle] [s(0), idle])$

added a collection of atomic propositions  $\Pi$  on its set of states. Intuitively, in this case of model checking, for each term  $t$  with variables (denoting a symbolic state) the truth value of the atomic propositions  $\Pi$  may not be defined without further instantiation of  $t$ . Therefore, we cannot perform only one narrowing step in order to build the symbolic Kripke structure and must perform a narrowing step  $t \rightsquigarrow_{R,E} t'$  composed with a new relation  $t' \rightsquigarrow_{\Pi} \sigma(t')$  that finds an appropriate substitution  $\sigma$  such that the truth value of the atomic propositions in  $\Pi$  is entirely defined for  $\sigma(t')$ .

In rewriting logic we usually specify a concurrent system as a topmost rewrite theory  $\mathcal{R} = (\Sigma, E, R)$ , and the atomic propositions  $\Pi$  as equationally-defined predicates in an equational theory  $\mathcal{E}_{\Pi} = (\Sigma_{\Pi}, E_{\Pi} \uplus E)$ . As explained in Section 3, the rewrite theory  $\mathcal{R}$  contains a top sort **State**, whose data elements are  $E$ -equivalence classes in  $\mathcal{T}_{\Sigma/E, \text{State}}$ , and rewrite rules  $l \rightarrow r \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\text{State}}$  denoting system transitions. We assume that  $\Sigma_{\Pi} = \Sigma \uplus \Pi \uplus \{\mathbf{tt}, \mathbf{ff}\}$ , where there is a new top sort **Bool** with no subsorts, containing only constants  $\mathbf{tt}$  and  $\mathbf{ff}$ , and each  $p \in \Pi$  is an atomic proposition function symbol  $p : \text{State} \rightarrow \text{Bool}$ . Furthermore, we assume that each equation in  $E_{\Pi}$  is of the form  $p(t) = \mathbf{tt}$  or  $p(t) = \mathbf{ff}$ , where  $p \in \Pi$  and  $t \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\text{State}}$ , and  $E_{\Pi}$  is sufficiently complete and protects **Bool** (for further details see [16]).

We define a  $\Pi$ -Kripke structure associated to a rewrite theory  $\mathcal{R}$  and an equational theory  $\mathcal{E}_{\Pi}$  defining the atomic propositions  $\Pi$  as the triple  $\mathcal{T}_{\mathcal{R}}^{\Pi} = (\mathcal{T}_{\Sigma/E, \text{State}}, (\rightarrow_{R,E})^{\bullet}, \mathcal{L}_{\Pi})$ , where for each  $[t]_E \in \mathcal{T}_{\Sigma/E, \text{State}}$  and  $p \in \Pi$ , we have  $p \in \mathcal{L}_{\Pi}([t]_E) \iff p(t) =_{(E_{\Pi} \uplus E)} \mathbf{tt}$ . In what follows we will always assume that  $\mathcal{R}$  is *deadlock free*, that is, that the set of  $\rightarrow_{R,E}$ -canonical forms of sort **State** is empty. As explained in [10,29], this involves no real loss of generality, since  $\mathcal{R}$  can always be transformed into a bisimilar  $\mathcal{R}^{\text{df}}$  which is deadlock free. Under this assumption the Kripke structure  $\mathcal{T}_{\mathcal{R}}^{\Pi}$  then becomes the pair  $\mathcal{T}_{\mathcal{R}}^{\Pi} = (\mathcal{T}_{\mathcal{R}}, \mathcal{L}_{\Pi})$ . As in Section 3, given a set  $U \subseteq \mathcal{T}_{\Sigma/E, \text{State}}$  of initial states, we abuse the notation and define the reachable sub  $\Pi$ -Kripke structure of  $\mathcal{T}_{\mathcal{R}}^{\Pi}$  by  $\mathcal{R}eac\mathcal{H}_{\mathcal{T}_{\mathcal{R}}^{\Pi}}(U)$ .



**Fig. 5.** Infinite Kripke structure  $\mathcal{R}eac\hbar_{\mathcal{T}_R^{\mathit{II}}}(0 ; 0 ; [0, \text{idle}])$

*Example 5.* Consider Example 1. We are interested in the atomic propositions  $\mathit{II} = \{\text{ever-wait?}, \text{ever-crit?}\}$  expressing that at least one process has been in its waiting (resp. critical) state.

```
fmod BAKERY-PROPS is
protecting BAKERY-SYNTAX .
sort Bool . ops tt ff : -> Bool .
ops ever-wait? ever-crit? : State -> Bool .
vars N M : Nat . vars PS : ProcSet .
eq ever-wait?(0 ; M ; PS) = ff .
eq ever-wait?(s(N) ; M ; PS) = tt .
eq ever-crit?(N ; 0 ; PS) = ff .
eq ever-crit?(N ; s(M) ; PS) = tt .
endfm
```

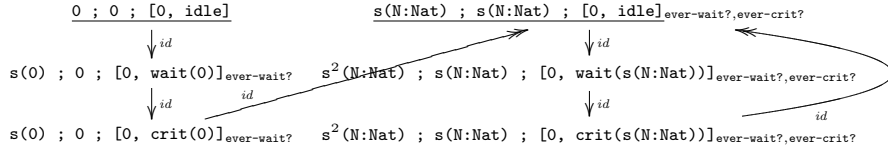
Given the initial state  $t_1 = \text{“}0 ; 0 ; [0, \text{idle}]\text{”}$ , the infinite  $\mathit{II}$ -Kripke structure  $\mathcal{R}eac\hbar_{\mathcal{T}_R^{\mathit{II}}}(t_1)$  is depicted in Figure 5, where we would like to verify the temporal formulas  $\text{“}\text{ever-wait?} \Rightarrow \Diamond \text{ever-crit?}\text{”}$  and  $\text{“}\Box(\text{ever-crit?} \Rightarrow \text{ever-wait?})\text{”}$ .

As explained above, we can have symbolic states (i.e., terms  $\mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^\circ$ ) such that the atomic propositions  $\mathit{II}$  cannot be evaluated without further instantiation; check the transition system of Figure 3, where propositions `ever-wait?` and `ever-crit?` cannot be evaluated in the node  $\text{“}N:\text{Nat} ; M:\text{Nat} ; [0, \text{MD:Mode}]\text{”}$ . We use the following relation that instantiates terms as least as possible to make propositions in  $\mathit{II}$  defined.

$$t \overset{\theta}{\rightsquigarrow}_{\mathit{II}} \theta(t) \iff \theta \in \text{CSU}_{(E_{\mathit{II}} \uplus E)}(p_1(t) = w_1 \wedge \dots \wedge p_n(t) = w_n)$$

where for each  $1 \leq i \leq n$ ,  $w_i$  is either `tt` or `ff`

This instantiation relation is based on whether there is a finitary and complete unification algorithm for the equational theory  $\mathcal{E}_{\mathit{II}}$ , which is satisfied by the equational theories used in this paper. We can exploit the generality of narrowing and define a Kripke-structure associated to narrowing based on the following set of terms  $\mathcal{T}_{\Sigma/E}^{\mathit{II}}(\mathcal{X})_{\text{State}}$  and the following relation  $\rightsquigarrow_{R,E;\mathit{II}}$ . We define the set of terms where the truth value of  $\mathit{II}$  is defined as  $\mathcal{T}_{\Sigma/E}^{\mathit{II}}(\mathcal{X})_{\text{State}} = \{t \in \mathcal{T}_{\Sigma/E}(\mathcal{X})_{\text{State}}^\circ \mid \forall p \in \mathit{II} : (p(t) =_{(E_{\mathit{II}} \uplus E)} \text{tt}) \vee (p(t) =_{(E_{\mathit{II}} \uplus E)} \text{ff})\}$ . The narrowing relation  $\rightsquigarrow_{R,E;\mathit{II}}$  is defined as  $\rightsquigarrow_{R,E}; \rightsquigarrow_{\mathit{II}}$ , i.e.,  $t \overset{\theta}{\rightsquigarrow}_{R,E;\mathit{II}} t'$  iff  $\exists w, \sigma, \sigma'$  s.t.  $t \overset{\sigma}{\rightsquigarrow}_{R,E} w$ ,  $w \overset{\sigma'}{\rightsquigarrow}_{\mathit{II}} t'$ , and  $\theta = \sigma \circ \sigma'$ . Note that  $\rightsquigarrow_{R,E;\mathit{II}}$  on  $\mathcal{T}_{\Sigma}(\mathcal{X})$  can be extended to a relation  $\overset{\sigma}{\rightsquigarrow}_{R,E;\mathit{II}}$  on  $\mathcal{T}_{\Sigma/E}(\mathcal{X})$  as  $(\overset{\sigma}{\rightsquigarrow}_{R,E;\mathit{II}}); (=E)$ . We define a Kripke-structure associated to narrowing as  $\mathcal{N}_{\mathcal{R}}^{\mathit{II}} = (\mathcal{T}_{\Sigma/E}^{\mathit{II}}(\mathcal{X})_{\text{State}}, \rightsquigarrow_{R,E;\mathit{II}}, \mathcal{L}_{\mathit{II}})$ , where for each  $[t]_E \in \mathcal{T}_{\Sigma/E}^{\mathit{II}}(\mathcal{X})_{\text{State}}$  and  $p \in \mathit{II}$ , we have  $p \in \mathcal{L}_{\mathit{II}}([t]_E) \iff p(t) =_{(E_{\mathit{II}} \uplus E)} \text{tt}$ .



**Fig. 6.** Finite Kripke structure  $\mathcal{R}eac\hbar_{\mathcal{N}^E}^E(\{w_1, w_2\})$  with  $w_1 =$  “0 ; 0 ; [0, idle]” and  $w_2 =$  “s(N:Nat) ; s(N:Nat) ; [0, idle]”

Results similar to Theorem 1, Lemma 2, Theorem 2, Theorem 3, Lemma 3, and Theorem 4 can be stated and proved for a deadlock-free topmost rewrite theory with a top sort **State** and a equational theory defining the atomic propositions. Such results and their proofs are included in [16].

*Example 6.* Consider Example 5. Consider the initial state  $w =$  “N:Nat ; N:Nat ; [0, idle]”, whose transition system is depicted in Figure 3. This transition system cannot be directly transformed into a  $\Pi$ -Kripke structure, since propositions  $\{\text{ever-wait?}, \text{ever-crit?}\}$  cannot be evaluated in, for instance, state “N:Nat ; N:Nat ; [0, idle]”. Therefore, we must for example instantiate term  $w$  using the narrowing relation  $\rightsquigarrow_{\Pi}$  and obtain terms  $w_1 =$  “0 ; 0 ; [0, idle]” and  $w_2 =$  “s(N:Nat) ; s(N:Nat) ; [0, idle]”, i.e.,  $w \rightsquigarrow_{\Pi} w_1$  and  $w \rightsquigarrow_{\Pi} w_2$ . The entire  $\Pi$ -Kripke structure  $\mathcal{R}eac\hbar_{\mathcal{N}^E}^E(\{w_1, w_2\})$  is depicted in Figure 6, where, since it is a finite-state system, we can use standard LTL model checking techniques to model check the formulas “ $\text{ever-wait?} \Rightarrow \Diamond \text{ever-crit?}$ ” and “ $\Box(\text{ever-crit?} \Rightarrow \text{ever-wait?})$ ”, which in this case hold in  $\mathcal{R}eac\hbar_{\mathcal{N}^E}^E(\{w_1, w_2\})$ . Therefore, the above LTL formulas also hold for the infinite-state system  $\mathcal{T}_{\mathcal{R}}^{\Pi}$  of Example 5 and the infinite set  $\llbracket \{w_1, w_2\} \rrbracket$  of initial states. Note that given that all substitutions in  $\mathcal{R}eac\hbar_{\mathcal{N}^E}^E(\{w_1, w_2\})$  are identity substitutions, we have a bisimulation and then CTL\* formulas can also be verified.

Similar arguments to those in Section 3 can be given in favor of narrowing for model checking  $ACTL^*$  (or  $CTL^*$ ) properties of systems that are either infinite-state or too big for standard finite-state methods. For example, when a set  $U \subseteq \mathcal{T}_{\Sigma/E}^{\Pi}(\mathcal{X})_{\text{State}}$  of initial states is provided,  $\mathcal{R}eac\hbar_{\mathcal{N}^E}^G(U)$  for some  $G$  such as  $\preceq_E$  can be finite when  $\mathcal{R}eac\hbar_{\mathcal{T}_{\mathcal{R}}^{\Pi}}(\llbracket U \rrbracket)$  is infinite, or can be much smaller even in the finite-state case. And  $U$  can be finite whereas  $\llbracket U \rrbracket$  may easily be infinite, making it impossible to verify properties by standard model checking algorithms.

## 5 Concluding Remarks

We have shown that, by specifying possibly infinite concurrent systems as rewrite theories, narrowing gives rise to a lifting simulation and provides a useful semi-decision procedure to answer reachability questions. We have also proposed a

method to fold the narrowing graph that, when it yields a finite system, allows algorithmic verification of such reachability questions, including invariants. Furthermore, we have extended these techniques to the verification of *ACTL\** and *LTL* formulas. Much work remains ahead, including: (i) gaining experience with many more examples such as concurrent systems, security protocols, Java program verification, etc.; (ii) implementing these techniques in *Maude*, taking advantage of its LTL model checker; (iii) investigating other folding relations that might further improve the generation of a finite narrowing search space; (iv) allowing more general state predicate definitions, for example with data parameters; (v) studying how grammar-based techniques and narrowing strategies can be used to further reduce the narrowing search space; and (vi) extending the results in this paper to more general temporal logics such as TLR [28].

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