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Abstract Model Checking of *tccp* programs

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Abstract

The *Timed Concurrent Constraint* programming language (*tccp*) introduces time aspects into the Concurrent Constraint paradigm. This makes *tccp* especially appropriate to analyze by model checking timing properties of concurrent systems. However, even if very compact state representations are obtained thanks to the use of constraints in *tccp*, large state spaces can be still generated which may prevent model checking tools from verifying *tccp* programs completely. In this paper, we introduce an abstract methodology which is based on over- and under-approximating *tccp* models and mitigates the state explosion problem which is common to traditional model checking algorithms. We ascertain the conditions for the correctness of the abstract technique and show that, due to the timing aspects of the language, this semantics does not correctly simulate the suspension behavior, which is a key feature of *tccp*. Then, we present a refined abstract semantics which correctly models suspension.

Keywords: Model Checking, Timed Concurrent Constraint Programming, Abstract Interpretation.

1 Introduction

In the past few years, some extensions of the concurrent constraint paradigm [4,20] have been defined in order to model reactive systems. All these extensions introduce a notion of time which makes it possible to model the typical

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ingredients of these systems, such as timeouts, preemptions, etc. The automatic verification of systems specified in the timed concurrent constraint language `tccp` of [4] was first studied in [11]. Then, an exhaustive method for applying the classical model checking technique to `tccp` was proposed in [12] which uses the temporal logic for reasoning about `tccp` programs of [5]. The main idea behind these methods is to take advantage of the constraint dimension of `tccp` in order to obtain a compact representation of the system which is then used as an input for the model checking algorithms. Unfortunately, both [11] and [12] develop exhaustive model checking algorithms, which causes the traditional state explosion problem and makes them not applicable to large size systems. In this work, we develop some suitable approximation techniques which are based on abstract interpretation [8] in order to drastically reduce the state space of model checking `tccp` thus providing a framework where exhaustive analysis of more complicated systems could be achieved.

Abstract model checking [7,10,17] combines abstract interpretation [8] and model checking [6] to improve the automatic verification of large systems. Applying abstract model checking involves the abstraction of both the model to be analyzed (M) and the properties to be checked within the model. Usually, in the classic abstract model checking literature, the abstract model M^+ is an over-approximation of the original one M , meaning that each possible concrete execution trace is mimicked in the abstract model. This approach allows one to verify properties which regard all the possible behavior paths. Two techniques have been successfully developed to construct M^+ . The *predicate abstraction* approach consists of substituting some selected model expressions with boolean variables, which leads to important simplifications (e.g., this is used in the tool SLAM [3,2]). In contrast, the *data abstraction* method reduces the type of certain data by transforming its original concrete domain into an approximate and simpler domain. For instance, this second approach has been used for abstracting models in the tools Bandera [16] and α SPIN [13].

In this paper, we follow the *data abstraction* method to approximate `tccp` computations. The common way of formalizing this technique is to introduce *abstract operations* that over-approximate the original ones (see, for instance, [14] where a data-based abstraction for the modeling language Promela is developed). However, inaccurate abstract models would be obtained by over-approximation due to the time dimension of `tccp`. We overcome this problem by combining over- and under-approximation in the abstraction of `tccp` operators. This approach is novel and allows us to build abstract models which are satisfactorily precise.

The abstraction of models usually involves adding non-determinism, due to the loss of information caused by the abstraction, and does affect the sus-

pension of processes: the suspension of a process in the original model does not generally imply that the process abstractly suspends. This is why abstraction is not adequate for analyzing system deadlocks. In the case of `tccp`, changing the suspension behavior might have undesirable consequences because processes are totally synchronized meaning that, at each time instant, all enabled agents (i.e., actions) involving a unit time are simultaneously carried out.

We have successively solved these two problems in the paper. First, we have defined an abstract semantics which takes into account the potential non-determinism added by the abstraction. We have proved that, provided the suspension behavior is correctly simulated, the abstract semantics is correct w.r.t. the original one. We have also defined a source-to-source transformation from the `tccp` program into its abstract version which is valid for this case. Source-to-source transformations are particularly interesting in the context of abstract model checking because they permit reusing model checkers. Next, we have slightly modified the abstract semantics, solving the suspension problem mentioned above. Due to lack of space, the problem of abstracting properties to adapt them to the new model representation is not dealt with in the paper.

The paper is organized as follows. Section 2 recalls the main features of the `tccp` language. In Section 3, we introduce our data abstraction methodology for `tccp`. We also present the abstract semantics for `tccp`, which does not take into account the potential suspension of processes. Section 4 discusses the correctness of this semantics and introduces a refined abstract semantics which correctly models process suspension. Finally, Section 5 shows the conclusions and discusses some future work.

2 The `tccp` language

In [4], the *Timed Concurrent Constraint* language (`tccp` in short) was defined as an extension of the Concurrent Constraint programming language `ccp` [19]. In the `cc` paradigm, the notion of *store as valuation* is replaced by the notion of *store as constraint*. The computational model is based on a global store where constraints are accumulated, and a set of agents which interact with the store. The model is parametric w.r.t. a cylindric constraint system \mathcal{C} defined below. In `tccp`, a new conditional agent (`now c then A else A`) is introduced (w.r.t. `ccp`) which makes it possible to model behaviors where the absence of information can cause the execution of a specific action. Intuitively, the execution of a `tccp` program evolves by asking and telling information to the store. Let us

briefly recall the syntax of the language:

$$A ::= \text{stop} \mid \text{tell}(c) \mid \sum_{i=1}^n \text{ask}(c_i) \rightarrow A_i \mid \text{now } c \text{ then } A \text{ else } A \mid A \parallel A \mid \exists x A \mid \mathfrak{p}(x)$$

where c, c_i are *finite constraints* (i.e., atomic propositions) of \mathcal{C} . A *tccp process* P is an object of the form $D.A$, where D is a set of procedure declarations of the form $\mathfrak{p}(x) ::= -A$, and A is an agent.⁶

Intuitively, the **stop** agent finishes the execution of the program, **tell**(c) adds the constraint c to the store, whereas the choice agent ($\sum_{i=1}^n \text{ask}(c_i) \rightarrow A_i$) consults the store and non-deterministically executes the agent A_i in the following time instant, provided the store satisfies the condition c_i ; otherwise the agent suspends. The conditional agent (**now** c **then** A **else** B) can detect *negative information* in the sense that, if the store satisfies c , then the agent A is executed; otherwise (even if $\neg c$ does not hold), B is executed. $A1 \parallel A2$ executes the two agents $A1$ and $A2$ in parallel. The $\exists x A$ agent is used to hide the information regarding x , i.e., it makes x local to the agent A .

The notion of time is introduced by defining a global clock which synchronizes all agents. In the semantics, the only agents which consume time (*timing agents*) are the tell, choice and procedure call agents.

We show an example of a tccp program in Figure 1. This program models a photocopier by means of four procedure declarations which represent the two main processes (**user**(C, A) and **photocopier**(C, A, Max, E, T)) and the synchronization of such processes (**system**(Max, E, C, A, T) and **initialize**(Max)).

Agent **user**(C, A) can execute non-deterministically three different actions or do nothing. The system is assumed to be synchronous, in the sense that the user cannot execute any action (through stream C) before the photocopier satisfies the previous request. This fact is modeled by the stream variable A .

The stream variable T is used to detect if no request has been received after Max time units. When this occurs, the photocopier is automatically turned-off. The system is initialized (**initialize**(Max)) and then synchronized by executing in parallel the photocopier and the user processes.

3 Abstracting tccp programs

Recently, some model checking algorithms have been formalized for the concurrent constraint paradigm. The common idea behind them is to exploit the constraint nature of the language to represent the model of the system in a compact way. However, the state explosion problem of classical model

⁶ We assume that all programs which we consider in this work are well typed.

```

user(C,A) ::= ask(A=[free|_]) → tell(C=[on|_]) +
  ask(A=[free|_]) → tell(C=[off|_]) +
  ask(A=[free|_]) → tell(C=[nc|_]) +
  ask(A=[free|_]) → tell(true).

photocopier(C,C',A,A',Max,E,E',T,T') ::= ∃ Aux,Aux' (
  now (T=[Aux|_] ∧ Aux>0) then
    now (C=[on|_]) then
      tell(E'=[going|_] ∧ T'=[Max|_] ∧ A'=[free|_])
    else now (C=[off|_]) then
      tell(E'=[stop|_] ∧ T'=[Max|_] ∧ A'=[free|_])
    else now (C=[nc|_]) then
      tell(E'=[going|_] ∧ T'=[Max|_] ∧ A'=[free|_])
      else tell(Aux'=Aux-1) || tell(T'=[Aux'|_] ∧ A'=[free|_])
    else tell(E'=[stop|_]) || tell(A'=[free|_])).

system(Max,E,C,A,T) ::= ∃ E',C',A',T' (tell(E=[_|E']) || tell(C=[_|C']) ||
  tell(A=[_|A']) || tell(T=[_|T']) || user(C,A) ||
  ask(true)→ask(true)→photocopier(C,C',A,A',Max,T,T',E,E') ||
  ask(A'=[free|_])→system(Max,E',C',A',T')).

initialize(Max) ::= ∃ E,C,A,T (tell(A=[free|_]) || tell(T=[Max|_]) ||
  tell(E=[off|_]) || system(Max,E,C,A,T)).

```

Figure 1. tccp program modeling a photocopier

checking techniques also happens in these algorithms. In order to avoid this problem, symbolic representations ([1,18]) and abstract models have been proposed ([10,14]), which we combine in this work.

3.1 Abstracting constraint systems

Definition 3.1 A *simple constraint system* is a structure such as $\langle \mathcal{C}, \vdash \rangle$ where \mathcal{C} is the set of atomic constraints and relation $\vdash \subseteq \wp(\mathcal{C}) \times \mathcal{C}$ satisfies

- C1. $u \vdash C$, for all $C \in u$. C2. $u \vdash C$, if $u \vdash C'$, $\forall C' \in v$, and $v \vdash C$

Relation \vdash may be extended to the relation $\vdash \subseteq \wp(\mathcal{C}) \times \wp(\mathcal{C})$ as:

$$u \vdash v \iff \forall C \in v, u \vdash C$$

Proposition 3.2 Relation \vdash has the following properties:

- (i) (*Reflexivity*) $\forall u \in \wp(\mathcal{C}). u \vdash u$.
(ii) (*Transitivity*) $\forall u, v, w \in \wp(\mathcal{C}). u \vdash v, v \vdash w$ implies that $u \vdash w$.

During `tccp` program computation, stores are represented by elements of $\wp(\mathcal{C})$. That is, if $u \subseteq \mathcal{C}$ is the current store, the information accumulated by u is the *conjunction* of all constraints $C \in u$. In addition, \vdash is the entailment relation used to deduce information from stores.

An *abstract interpretation* (an *abstraction*) of the simple constraint system (\mathcal{C}, \vdash) is given by defining an *upper closure operator* (uco) $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, that is, a *monotonic* ($sst_1 \subseteq sst_2$ then $\rho(sst_1) \subseteq \rho(sst_2)$), *idempotent* ($\rho(sst) = \rho(\rho(sst))$) and *extensive* ($sst \subseteq \rho(sst)$) operator. The intuition of this definition is that each store $st \subseteq \mathcal{C}$ is abstracted by its closure $\rho(\{st\})$. Closure operators have many interesting properties. For instance, when the domain considered is a complete lattice, as $(\wp(\wp(\mathcal{C})), \subseteq)$, each closure operator is uniquely determined by the set of its fixed points. In the context of abstract interpretation, closure operators are important because abstract domains can be equivalently defined by using them or by Galois insertions as introduced in [9]. Let $\iota : \wp(\wp(\mathcal{C})) \rightarrow E$ be an isomorphism. Then, given an uco $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, structure $(\wp(\wp(\mathcal{C})), \iota \circ \rho, \iota^{-1}, E)$ is a Galois insertion, where $\iota \circ \rho$ and ι^{-1} are the abstraction and concretization functions, respectively.

Note that, using abstract interpretation terminology, $\rho(\{st\})$ is the most precise abstraction of the store $st \in \wp(\mathcal{C})$ and if $\rho(\{st\}) \subseteq sst$, then sst is also an abstraction of st .

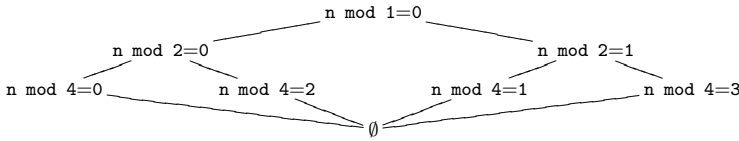


Figure 2. Closure operator ρ_x

Example 3.3 Let $\mathcal{C} = \{x = n, y = m | n, m \in \mathbb{N}\}$, and $\rho_x : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$ an abstraction which does not affect variable y , while the abstract value of x is given in Figure 2. In this figure, expression $n \bmod a = b$ represents the set of stores which contain the constraint $x = n$, with $n \bmod a = b$. In order to construct ρ_x , we define the following sets of stores where $m \in \mathbb{N}$: $[y = m] = \{\{y = m\}\}$, $[n \bmod a = b, y = m] = \{\{x = b + ak, y = m\} | k \in \mathbb{N}\}$ and $[n \bmod a = b] = \{\{x = b + ak\} | k \in \mathbb{N}\}$

Using the lub operator induced by the lattice shown in Figure 2 (denoted below as \sqcup_x), we define operator \sqcup_x over these sets as:

- $[n \bmod a_1 = b_1, y = m] \sqcup_x [n \bmod a_2 = b_2, y = m] = [(n \bmod a_1 = b_1) \sqcup (n \bmod a_2 = b_2), y = m]$.
- $[n \bmod a_1 = b_1] \sqcup_x [n \bmod a_2 = b_2] = [(n \bmod a_1 = b_1) \sqcup (n \bmod a_2 = b_2)]$.
- $[c_1] \sqcup_x [c_2] = [c_1] \cup [c_2]$, otherwise.

Now, ρ_x is defined as: $\rho_x(\emptyset) = \emptyset$; $\rho_x(\{st\}) = [c]$ iff $[c]$ is the smallest set of stores

such that $st \in [c]; \rho_x(\{st_i | i \in I\}) = (\bigsqcup_x)_{i \in I} \rho_x(\{st_i\})$.

The following definition introduces two dual entailment relations into the abstract constraint systems.

Definition 3.4 Given a simple constraint system $\langle \mathcal{C}, \vdash \rangle$ and an abstraction $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, we define the *over-* and *under-approximated constraint systems* $\langle \wp(\mathcal{C}), \vdash_\rho^+ \rangle$ and $\langle \wp(\mathcal{C}), \vdash_\rho^- \rangle$ where $\vdash_\rho^+, \vdash_\rho^- \subseteq \wp(\wp(\mathcal{C})) \times \wp(\wp(\mathcal{C}))$ are:

- $sst_1 \vdash_\rho^+ sst_2 \iff \exists u \in \rho(sst_1), \exists v \in sst_2$ such that $u \vdash v$.
- $sst_1 \vdash_\rho^- sst_2 \iff \forall u \in \rho(sst_1), \exists v \in sst_2, u \vdash v$.

The following proposition justifies the names of the new structures given in the previous definition.

Proposition 3.5 *Given a simple constraint system $\langle \mathcal{C}, \vdash \rangle$ and an abstraction $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, then*

1. *If $u \vdash v$, then $\rho(\{u\}) \vdash_\rho^+ \{v\}$.*
2. *If $\rho(\{u\}) \vdash_\rho^- \{v\}$, then $u \vdash v$*

Proposition 3.6 *Given a simple constraint system $\langle \mathcal{C}, \vdash \rangle$ and an abstraction $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, then*

- (i) *(Reflexivity for \vdash_ρ^+) $\forall sst \in \wp(\wp(\mathcal{C})). sst \vdash_\rho^+ sst$.*
- (ii) *(Transitivity for \vdash_ρ^-) $\forall sst_1, sst_2, sst_3 \in \wp(\wp(\mathcal{C})). sst_1 \vdash_\rho^- sst_2, sst_2 \vdash_\rho^- sst_3$ implies that $sst_1 \vdash_\rho^- sst_3$.*

Intuitively, the image associated to a store by means of \vdash_ρ^+ is bigger than the one obtained by applying \vdash_ρ^- to the same store. However, it is worth noting that, in general, relation \vdash_ρ^+ is not transitive and \vdash_ρ^- is not reflexive, as shown in the following example. This means that abstract entailment relations lose information with respect to the original relation as usual when abstraction is applied. In addition, each abstract relation loses only one of the two properties characterizing the entailment relations. This fact follows from the definition of the relations.

Example 3.7 Assume that the constraint system of Example 3.3 is extended with the constraint $n \bmod 2 = 0$ containing the natural meaning about value n of variable x .

- (i) \vdash_ρ^+ is not transitive. $\{\{x = 8\}\} \vdash_{\rho_x}^+ \{\{n \bmod 2 = 0\}\}$ and $\{\{n \bmod 2 = 0\}\} \vdash_{\rho_x}^+ \{\{x = 6\}\}$, since $\{x = 6\} \in \rho_x(\{\{n \bmod 2 = 0\}\})$ and $\{x = 6\} \vdash \{x = 6\}$. However, $\{\{x = 8\}\} \not\vdash_{\rho_x}^+ \{\{x = 6\}\}$.
- (ii) \vdash_ρ^- is not reflexive. $\{\{x = 2\}\} \not\vdash_{\rho_x}^- \{\{x = 2\}\}$, since $\{x = 6\} \in \rho_x(\{\{x = 2\}\})$ and $\{x = 6\} \not\vdash \{x = 2\}$.

Definition 3.8 Let us define the operator $\sqcup^\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$ as $sst_1 \sqcup^\rho sst_2 = \rho(\{u \cup v \mid u \in sst_1, v \in sst_2, u \cup v \vdash true\})$.

Operator \sqcup^ρ provides the abstract information given by the accumulation of the constraints in its two operands. Note that, following the operational semantics of the language, inconsistent stores might appear (due to the tell agent), thus it is necessary to explicitly remove them from $sst_1 \sqcup^\rho sst_2$ as is done by means of the operator \sqcup^ρ . The following proposition states that operator \sqcup^ρ correctly approximates \cup .

Proposition 3.9 For all $u, v \in \wp(\mathcal{C})$ if $u \cup v \vdash true$ then $\rho(\{u \cup v\}) \subseteq \rho(\{u\}) \sqcup^\rho \rho(\{v\})$

In *tccp*, special constraint systems called *cylindric constraint systems* are used. Cylindric constraint systems are defined as follows.

Definition 3.10 $\langle \mathcal{C}, \vdash, Var, \exists \rangle$ is a *cylindric constraint system* iff $\langle \mathcal{C}, \vdash \rangle$ is a simple constraint system, Var is a denumerable set of variables, and for each $x \in Var$, there exists a function $\exists_x : \wp(\mathcal{C}) \rightarrow \wp(\mathcal{C})$ such that for each $u, v \in \wp(\mathcal{C})$:

- | | |
|---|--|
| (i) $u \vdash \exists_x u$ | (iii) $\exists_x(u \cup \exists_x v) = \exists_x u \cup \exists_x v$ |
| (ii) $u \vdash v$ then $\exists_x u \vdash \exists_x v$ | (iv) $\exists_x(\exists_y u) = \exists_y(\exists_x u)$ |

A *set of diagonal elements* for a cylindric constraint system is a family $\{\delta_{xy} \in \mathcal{C} \mid x, y \in var\}$ such that

- | | |
|--|--|
| (i) $\emptyset \vdash \delta_{xx}$ | (iii) If $x \neq y$ then $\delta_{xy} \cup \exists_x(v \cup \delta_{xy}) \vdash v$. |
| (ii) If $y \neq x, z$ then $\delta_{xz} = \exists_x(\delta_{xy} \cup \delta_{yz})$. | |

Diagonal elements allow us to hide variables, representing local variables, as well as to implement parameter passing among predicates. Thus, quantifier \exists_x and diagonal elements δ_{xy} allow us to properly deal with variables in constraint systems. Assuming that the original constraint system $\langle \mathcal{C}, \vdash \rangle$ to be abstracted is a cylindric constraint system, and given an abstraction $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, in general, the over and under-approximated constraint systems $\langle \wp(\mathcal{C}), \vdash_\rho^+ \rangle$ and $\langle \wp(\mathcal{C}), \vdash_\rho^- \rangle$ are not cylindric constraint systems. Example 3.7 shows that some property of the underlying simple constraint system may be lost during the abstraction process. In addition, the remaining properties concerning the existential quantifier or the diagonal elements may also be lost. An extensive study of the conditions that the abstraction ρ has to satisfy for the abstraction process to preserve all these properties can be found in [15] where a generalized semantics for logic concurrent languages is introduced. In short, ρ must satisfy some consistency properties to ensure

that the existential quantification behaves correctly. We extend function \exists_x to sets of stores as $\exists_x : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$ where $\exists_x sst = \{\exists_x u \mid u \in sst\}$.

Example 3.11 Consider the `tccp` program modeling a photocopier shown in Figure 1. Let \mathcal{C} be the set of atomic constraints appearing in the code. Define the set $msg = \{on, off, nc\}$. Given $X, X' \in Var$, construct the sets $msg(X, X') = \{X = [A|X'] \mid A \in msg\}$ and $MSG = \cup_{X, X' \in Var} msg(X, X')$. Divide each store $u \in \wp(\mathcal{C})$ into the subsets $u_1 = u - MSG$ and $u_2 = u \cap MSG$. Then the upper closure operator $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$ is defined as follows.

- Given $u \in \wp(\mathcal{C})$, then $u' \in \rho(\{u\})$ if $u' = u_1 \cup u'_2$ where u'_2 is constructed by choosing an element $C' \in msg(X, X')$ whenever $msg(X, X') \cap u \neq \emptyset$.
- Given $u \in \wp(\mathcal{C})$, then $\rho(\{u\}) = \{u\}$ iff $\forall X, X' \in Var, msg(X, X') \cap u = \emptyset$.
- $\rho(sst) = (\cup_{u \in sst} \rho(\{u\}))$.

In short, function ρ abstracts the messages in msg . For instance,

$$\rho(\{\{X = [on|X']\}\}) = \{\{X = [off|X']\}, \{X = [on|X']\}, \{X = [nc|X']\}\}$$

Note that an implementation of this abstraction would substitute the concrete constants *on*, *off* and *nc* by an abstract constant (for example, *msg*), thus making the abstract store simpler.

3.2 Abstract Semantics

The source-to-source transformation from the original model into the abstract one is a well-known technique of integrating abstraction and model checking [13,16] since it enables reusing the existing model checkers for the original language. In this section, we study the difficulties of applying this method to `tccp` programs.

In the case of `tccp`, the abstraction of `now` has to be done with special care. The reason for this is that the non-determinism introduced when abstracting this agent cannot be handled in `tccp` instantaneously. The use of `ask` involves passing one time unit. To solve this problem, we have introduced a new agent `askα` which allows us to introduce non-determinism without consuming time.

We formalize the abstract operational semantics of a `tccp` model (with the new agent) in terms of a transition relation similar to the operational semantics of the original `tccp` language. Similarly to the original operational semantics of `tccp`, each transition involves time passing. However, we will show that the time aspects of `tccp` may impede the correct simulation of the synchronization of agents in the concrete model. This aspect differentiates `tccp` from other modeling languages having no time aspects.

R1	$\langle \text{stop}, sst \rangle \not\rightarrow$	R2	$\langle \text{tell}(c), sst \rangle \longrightarrow \langle \emptyset, sst \sqcup c \rangle$
R3	$\langle \sum_{i=0}^n \text{ask}(c_i) \rightarrow A_i, sst \rangle \longrightarrow \langle A_j, sst \rangle$ if $0 \leq j \leq n$, and $sst \vdash^+ c_j$		
R4	$\frac{\langle A_j, sst \rangle \longrightarrow \langle A'_j, sst' \rangle}{\langle \sum_i^n \text{ask}^\alpha(c_i) \rightarrow A_i, sst \rangle \longrightarrow \langle A'_j, sst' \rangle}$ if $sst \vdash^+ c_j$		
R5	$\frac{\langle B1, sst \rangle \longrightarrow \langle B1', sst' \rangle}{\langle \text{now } c \text{ then } B1 \text{ else } B2, sst \rangle \longrightarrow \langle B1', sst' \rangle}$ if $sst \vdash^- c$		
R6	$\frac{\langle B1, sst \rangle \not\rightarrow}{\langle \text{now } c \text{ then } B1 \text{ else } B2, sst \rangle \longrightarrow \langle B1, sst \rangle}$ if $sst \vdash^- c$		
R7	$\frac{\langle B2, sst \rangle \longrightarrow \langle B2', sst' \rangle}{\langle \text{now } c \text{ then } B1 \text{ else } B2, sst \rangle \longrightarrow \langle B2', sst' \rangle}$ if $sst \not\vdash^- c$		
R8	$\frac{\langle B2, sst \rangle \not\rightarrow}{\langle \text{now } c \text{ then } B1 \text{ else } B2, sst \rangle \longrightarrow \langle B2, sst \rangle}$ if $sst \not\vdash^- c$		
R9	$\frac{\langle B1, sst \rangle \longrightarrow \langle B1', sst'_1 \rangle, \langle B2, sst \rangle \longrightarrow \langle B2', sst'_2 \rangle}{\langle B1 B2, sst \rangle \longrightarrow \langle B1' B2', sst'_1 \sqcup sst'_2 \rangle}$		
R10	$\frac{\langle B1, sst \rangle \longrightarrow \langle B1', sst' \rangle, \langle B2, sst \rangle \not\rightarrow}{\langle B1 B2, sst \rangle \longrightarrow \langle B1' B2, sst' \rangle}$		
R11	$\frac{\langle B1, sst \rangle \not\rightarrow, \langle B2, sst \rangle \longrightarrow \langle B2', sst' \rangle}{\langle B1 B2, sst \rangle \longrightarrow \langle B1 B2', sst' \rangle}$		
R12	$\frac{\langle B, sst_1 \cup \exists x sst_2 \rangle \longrightarrow \langle B', sst' \rangle}{\langle \exists^{sst_1} x B, sst_2 \rangle \longrightarrow \langle \exists^{sst'} x B', sst_2 \sqcup \exists x sst' \rangle}$		
R13	$\langle p(x), sst \rangle \longrightarrow \langle B, sst \rangle$ if $p(x) ::= B \in D$		

Figure 3. Abstract operational semantics of tccp

In the semantic rules, we are assuming that an abstraction operator $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$ has been provided and it has the properties discussed in Section 3.1. We have dropped the subindex ρ from \vdash^+ , \vdash^- and \sqcup in order to simplify the presentation. For the same reason, in the sequel, we write $sst \vdash^+ c$, $sst \vdash^- c$ and $sst \sqcup c$ for $sst \vdash^+ \{\{c\}\}$, $sst \vdash^- \{\{c\}\}$ and $sst \sqcup \{\{c\}\}$.

The two main points of the abstract semantics are the new ask^α agent and the abstraction of the conditional agent. We show the transition rules for each agent in Figure 3.⁷ A *configuration* of the form $\langle \Gamma, sst \rangle$ represents a computation state, where Γ is a multiset of agents and $sst \in \wp(\wp(\mathcal{C}))$ is an abstract store.

Let us now explain the main differences between the abstract semantics and the concrete one defined in [4]. The first important point to observe in the semantics is the use of the two abstract entailment relations \vdash^+ and \vdash^- . The

⁷ In rule **R12**, the superscript in $\exists^d B$ represents the information accumulated during the execution of the agent B . See [4] for details.

application of the different rules in Figure 3 depends on these two abstract entailment relations. In particular, under-approximation is used only for the conditional agent. There is a completely new rule (**R4**), which defines the semantics for the instantaneous choice agent (ask^α). This rule states that, provided agent A_j can evolve to agent A'_j , the instantaneous choice can evolve to A'_j . It is important to remark the timing difference between rule **R3** and rule **R4**. Both of them introduce non-determinism but a time unit is consumed in the first one before executing the agent in the body of the ask agent, whereas in the second rule, non-determinism is introduced instantaneously.

3.3 Program Abstraction

In this section, we give a first step towards a source-to-source transformation of `tccp` programs into abstract programs which represent an approximate model of the system. For each `tccp` agent A , we inductively construct a corresponding abstract `tccp` agent $\alpha(A)$ as is shown in Figure 4.

<p>Stop agent. $\alpha(\text{stop}) = \text{stop}$. Tell agent. $\alpha(\text{tell}(c)) = \text{tell}(c)$.</p> <p>Choice agent. $\alpha(\sum_{i=0}^n \text{ask}(c_i) \rightarrow B_i) = \sum_{i=0}^n \text{ask}(c_i) \rightarrow \alpha(B_i)$.</p> <p>Conditional agent. $\alpha(\text{now } c \text{ then } B1 \text{ else } B2) =$ $\text{now } c \text{ then } \alpha(B1) \text{ else } \text{ask}^\alpha(c) \rightarrow \alpha(B1) + \text{ask}^\alpha(\text{true}) \rightarrow \alpha(B2)$</p> <p>Parallel agent. $\alpha(B1 B2) = \alpha(B1) \alpha(B2)$.</p> <p>Hiding agent. $\alpha(\exists x B) = \exists x \alpha(B)$, where x is a variable.</p> <p>Procedure Call agent. $\alpha(p(x)) = p(x)$ where x is a variable of the constraint system and there exists a procedure declaration as $p(x) ::= B$.</p>
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Figure 4. α -transformation for `tccp` programs

The intuitive idea of the transformation of the conditional agent is as follows. We let \vdash^- (\vdash^+) represent a suitable under (over) approximation of the entailment relation \vdash of the constraint system. In order to represent the possible conditional execution in the concrete model by an execution in the corresponding abstract model, we consider the following four possible cases, where $st \in \wp(\mathcal{C})$ and $sst \in \wp(\wp(\mathcal{C}))$ are, respectively, the concrete store and the abstract one, and $\rho(\{st\}) \subseteq sst$.

- if $st \vdash c$ and $sst \vdash^- c$, then $B1$ is executed in both the concrete and the abstract models,

- if $st \vdash c$ and $sst \not\vdash^- c$, then agent $B1$ is executed in the concrete model whereas any of the agents $B1$ or $B2$ could be executed in the abstract one,
- if $st \not\vdash c$ but $sst \vdash^+ c$, then in the concrete model agent $B2$ is executed, whereas any of the agents $B1$ or $B2$ could be executed in the abstract one,
- if $st \not\vdash c$ and $sst \not\vdash^+ c$, then both the abstract and the concrete models execute agent $B2$.

Note that the combination of the two abstract entailment relations allows us to precisely approximate the behavior of **now** for the first case above, whereas this could not be obtained by using only \vdash^+ .

4 Correctness

In this section, we demonstrate that some additional conditions concerning the suspension behavior of the program are needed for the abstract semantics of **tccp** programs to correctly approximate the standard one. In the case these conditions hold, we develop an abstraction-by-transformation method which allows us to approximate **tccp** computations. Finally, we show how the abstract semantics can be refined in order to preserve suspension. However, the transformation method for implementing abstraction can not be directly applied to the refined abstract semantics anymore.

4.1 Conditions to obtain correctness

Given a **tccp** program (a process) P of the form $D.\Gamma_0$ and an initial configuration $\langle \Gamma_0, st_0 \rangle$, a *trace* t of P starting at $\langle \Gamma_0, st_0 \rangle$ is a sequence of configurations $t = \langle \Gamma_0, st_0 \rangle \longrightarrow \dots$ built using the transition rules given by the operational semantics. Let $\mathcal{O}(P)(\langle \Gamma_0, st_0 \rangle)$ denote the set of traces generated using the standard operational semantics given in [4]. We say that a concrete trace $t = \langle \Gamma_0, st_0 \rangle \longrightarrow \dots \in \mathcal{O}(P)(\langle \Gamma_0, st_0 \rangle)$ is *erroneous* iff $\exists i \geq 0. st_i$ is not consistent.

Similarly, given an abstraction ρ , let $\mathcal{A}_\rho(P^\alpha)(\langle \Gamma_0, sst_0 \rangle)$ denote the set of abstract traces generated by the abstract program P^α using the abstract operational semantics given by Figure 3. Note that abstract program P^α may include the new agent ask^α .

Given a trace $t = \langle \Gamma_0, st_0 \rangle \longrightarrow \langle \Gamma_1, st_1 \rangle \longrightarrow \dots \in \mathcal{O}(P)(\langle \Gamma_0, st_0 \rangle)$, we denote with $\alpha(t)$ the abstract trace obtained by applying the transformation α presented previously to the agents in the configurations of t , and abstracting the corresponding stores using ρ , that is, $\alpha(t) = \langle \alpha(\Gamma_0), \rho(\{st_0\}) \rangle \longrightarrow \langle \alpha(\Gamma_1), \rho(\{st_1\}) \rangle \longrightarrow \dots$. In addition, given two abstract traces $t_1^\alpha = \langle \Gamma_0^\alpha, sst_{01} \rangle \longrightarrow \langle \Gamma_1^\alpha, sst_{11} \rangle \longrightarrow \dots$ and $t_2^\alpha = \langle \Gamma_0^\alpha, sst_{02} \rangle \longrightarrow \langle \Gamma_1^\alpha, sst_{12} \rangle \longrightarrow \dots$, we write $t_1^\alpha \sqsubseteq t_2^\alpha$ when $\forall i \geq 0. sst_{i1} \subseteq sst_{i2}$.

Conditions for Correctness (CC) Assume that function ρ preserves the local suspension from the concrete configurations to the abstract ones, that is, for all configuration Γ and for each store st , if $\langle \Gamma, st \rangle \not\rightarrow$ and $\rho(\{st\}) \subseteq sst$ then $\langle \alpha(\Gamma), sst \rangle \not\rightarrow$.

Theorem 4.1 *Given a tccp program P , an initial configuration $\langle \Gamma_0, st_0 \rangle$ and an abstraction function $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$ satisfying condition **CC**, then for each non erroneous trace $t \in \mathcal{O}(P)(\langle \Gamma_0, st_0 \rangle)$ there exists an abstract trace $t^\alpha \in \mathcal{A}_\rho(\alpha(P))(\langle \alpha(\Gamma_0), \rho(\{st_0\}) \rangle)$ such that $\alpha(t) \sqsubseteq t^\alpha$.*

Example 4.2 The abstraction given in Example 3.11 of the tccp program illustrated in Figure 1 satisfies **CC**. Note that the concrete model never suspends if stream C has a message, and the same applies to the abstract model. In addition, if C has no message, both the concrete and the abstract model suspend. Therefore, Theorem 4.1 may be applied in this case. This abstraction is useful to check properties like “the photocopier is switched off when it is inactive during *Max* time units”.

The abstraction may modify some time aspects, in such a way that abstract agents are not correctly synchronized, as illustrated by the following example.

Example 4.3 Consider the abstraction given by Figure 2 which takes into account the divisibility by four of variable X (recall that the abstract store $n \bmod 4 = 0$ represents the stores where $X = n$ and n satisfies this condition). Figure 5 shows that if **CC** does not hold, the abstract model does not correctly simulate the original one. In the figure, the new agents **A'** and **B'** appear since **A** and **B** could evolve to these agents when rules **R5** or **R7** are applied.

4.2 Implementation of the abstract semantics

In Section 3 we showed how it is possible to abstract tccp programs. An abstract semantics for the abstract model and a program transformation of the program was formulated. In this section, we show how we can implement the abstract semantics by using only the original tccp agents, i.e., eliminating the ask^α agent thus obtaining a source-to-source transformation from the concrete to the abstract model. We also prove that the implementation corresponds to the intended behavior of the abstract semantics.

We first recall the transformation of the conditional agent:

$$\alpha(\text{now } c \text{ then } B1 \text{ else } B2) = \text{now } c \text{ then } \alpha(B1) \text{ else } \text{ask}^\alpha(c) \rightarrow \alpha(B1) + \text{ask}^\alpha(\text{true}) \rightarrow \alpha(B2)$$

If we simply substitute the ask^α agent by the original ask one, then the body agent in the abstract model is executed in the current time instant, whereas in the new version it is executed in the next one. This means that

Concrete Trace	
STORE	AGENTS
X=0	ask(X=4) → tell(Y=2) ask(true) → ask(true) → now Y=2 then A else B
X=0	ask(X=4) → tell(Y=2) ask(true) → now Y=2 then A else B
X=0	ask(X=4) → tell(Y=2) now Y=2 then A else B
X=0	ask(X=4) → tell(Y=2) B'

Abstract trace	
STORE	AGENTS
n mod 4 = 0	ask(X=4) → tell(Y=2) ask(true) → ask(true) → now Y=2 then A else B ask(true) → ask(true) → now Y=2 then A else B
n mod 4 = 0	tell(Y=2) ask(true) → now Y=2 then A else B
n mod 4 = 0 ∧ Y=2	now Y=2 then A else B
n mod 4 = 0 ∧ Y=2	A'

Figure 5. A non correct abstract model

other agents that could eventually be executed concurrently could introduce information in the store which interferes with the execution, thus changing the program semantics.

In order to overcome this problem, we strategically introduce delays in the program. For this purpose, we first apply a static analysis to the program which determines what is the maximum number (N) of nested conditional agents in the program, and associates to each occurrence of timing agents an integer number which represents its relative depth (with respect to nested conditional agents). A_ϵ denotes that the relative depth of A is ϵ .

In short, the transformed program is obtained by introducing $N - \epsilon$ delays for each A_ϵ agent in the program. Note that it is also necessary to introduce the corresponding delay in the case when the choice agent suspends. This implies a more elaborated transformation which we do not present in this work due to the limited space. In Figure 6 we show the transformation associated to each agent. Notation $(\text{ask}(\mathbf{true}) \rightarrow)^{N-\epsilon}$ means that the $\text{ask}(\mathbf{true})$ agent is replicated $N - \epsilon$ times.

Intuitively, we can imagine that we split each time instant into N parts. When we abstract a concrete program, the structure of the abstracted program ensures that all agents corresponding to concrete agents (except for the conditional agent) are executed in the last part of each N block. Furthermore, the execution of conditional agents will start during the corresponding part of the block depending on its relative depth.

Given a program P , we call P^β to the β -program transformation. Next we prove that the β -transformation described above is correct. We say that

<p>Stop agent. $\beta(\text{stop}_\epsilon) = (\text{ask}(\mathbf{true}) \rightarrow)^{N-\epsilon} \text{stop}$.</p> <p>Tell agent. $\beta(\text{tell}(c)_\epsilon) = (\text{ask}(\mathbf{true}) \rightarrow)^{N-\epsilon} \text{tell}(c)$.</p> <p>Choice agent. $\beta(\sum_{i=0}^n \text{ask}(c_i)_\epsilon \rightarrow B_i) = (\text{ask}(\mathbf{true}) \rightarrow)^{N-\epsilon} \sum_{i=0}^n \text{ask}(c_i) \rightarrow \beta(B_i)$.</p> <p>Conditional agent. $\beta(\text{now } c \text{ then } B1 \text{ else } B2) =$ $\text{now } c \text{ then } \beta(B1) \text{ else } (\text{ask}(c) \rightarrow \beta(B1) + \text{ask}(\mathbf{true}) \rightarrow \beta(B2))$</p> <p>Parallel agent. $\beta(B1 B2) = (\beta(B1) \beta(B2))$.</p> <p>Hiding agent. $\beta(\exists x B) = \exists x \beta(B)$, where x is a variable.</p> <p>Procedure Call agent. $\beta(p(x)_\epsilon) = (\text{ask}(\mathbf{true}) \rightarrow)^{N-\epsilon} p(x)$ where x is a variable of the constraint system and there exists a procedure declaration as $p(x) ::= B$.</p>

Figure 6. β -transformation of tccp programs

the transformation is correct if, for all P , P^β simulates the behavior of P^α obtained by applying the α -transformation described in Section 3.

We define the notion of observable for β -programs as $NOb(P^\beta)(\langle \Gamma_0, sst_0 \rangle) = \{ \langle \Gamma_{0*N}, sst_{0*N} \rangle \longrightarrow \langle \Gamma_{1*N}, sst_{1*N} \rangle \longrightarrow \langle \Gamma_{2*N}, sst_{2*N} \rangle \longrightarrow \dots \mid \langle \Gamma_0, sst_0 \rangle \longrightarrow \langle \Gamma_1, sst_1 \rangle \longrightarrow \langle \Gamma_2, sst_2 \rangle \dots \in \mathcal{A}_\rho(P^\beta)(\langle \Gamma_0, sst_0 \rangle) \}$

Theorem 4.4 *Given a tccp program P , an initial configuration $\langle \Gamma_0, sst_0 \rangle$ and an abstraction function $\rho : \wp(\mathcal{C}) \rightarrow \wp(\mathcal{C})$ then*

$$NOb(P^\beta)(\langle \Gamma_0, sst_0 \rangle) = \mathcal{A}_\rho(P^\alpha)(\langle \Gamma_0, sst_0 \rangle)$$

The proof is based on the demonstration that the β -abstraction models the α -abstraction (and vice-versa), and is done by induction on both the length of traces (for the procedure call case) and the structure of agents.

4.3 A Correct Abstract Semantics

We finalize the discussion about the correct simulation of a tccp program by giving an abstract semantics where the problem shown by Figure 5 is solved. The idea is to non-deterministically allow the repetition of an abstract configuration when it is possible that some concretization may suspend. Note that again, in this case, the use of the abstract entailment relation \vdash^- makes it possible to precisely represent this situation.

Consider the transition system obtained by modifying the abstract semantics given in Figure 3 with the new rules presented by Figure 7, as follows. Rule **R1** is substituted by rule **R1'**, rule **R3'** is added, and rules **R6**, **R8**, **R10** and **R11** are dropped. Note that a configuration containing an **ask** is replicated with the new abstract semantics when it may suspend in the concrete semantics. This is how to simulate suspension in the abstract semantics.

R1'	$\langle \text{stop}, d \rangle \longrightarrow \langle \text{stop}, d \rangle$
R3'	$\langle \sum_{i=0}^n \text{ask}(c_i) \rightarrow A_i, d \rangle \longrightarrow \langle \sum_{i=0}^n \text{ask}(c_i) \rightarrow A_i, d \rangle$ if $\forall j. 0 \leq j \leq n, d \not\vdash^- c_j$

Figure 7. New rules for a correct abstract semantics of `tccp`

The new semantics (\mathcal{A}'_ρ) gives us the desired result about correctness.

Theorem 4.5 *Given a `tccp` program P of the form $D.\Gamma_0$, an initial configuration $\langle \Gamma_0, st_0 \rangle$ and an abstraction function $\rho : \wp(\wp(\mathcal{C})) \rightarrow \wp(\wp(\mathcal{C}))$, then for each non erroneous trace $t \in \mathcal{O}(P)(\langle \Gamma_0, st_0 \rangle)$ there exists an abstract trace $t^\alpha \in \mathcal{A}'_\rho(\alpha(P))(\langle \alpha(\Gamma_0), \rho(\{st_0\}) \rangle)$ such that $\alpha(t) \sqsubseteq t^\alpha$.*

Example 4.6 For instance, consider the traces given in Figure 5, with the new semantics we obtain the abstract simulation shown in Figure 8

New Abstract trace	
STORE	AGENTS
X mod 4 = 0	ask(X=4) → tell(Y=2) ask(true) → ask(true) → now Y=2 then A else B
X mod 4 = 0	ask(X=4) → tell(Y=2) ask(true) → now Y=2 then A else B
X mod 4 = 0	ask(X=4) → tell(Y=2) now Y=2 then A else B
X mod 4 = 0	ask(X=4) → tell(Y=2) B'

Figure 8. A correct abstract model

5 Conclusions and Future Work

In this work, we have proposed a methodology which mitigates the state explosion problem in `tccp` model checking. We have defined a source-to-source transformation for `tccp` programs which is based on over- and under-approximating constraint systems. The abstraction of the conditional `tccp` agent introduces some specific difficulties which have been solved by using under-approximation in the abstract semantics; this idea is novel since only over-approximations are typically used when approximating models in the data abstraction approach. The inspiration to combine over- and under-approximation in model-checking `tccp` comes from [13]. Moreover, we have proved that correctness of the semantics is not guaranteed unless several conditions concerning the suspension of agents do hold. We also provide a simple refinement of the abstract semantics which overcomes this problem. Unfortunately, we obtain a computation model which is not the one of `tccp` anymore. As future work, we plan to complete our methodology by defining an approximation technique for the

properties that must be verified. We intend to formulate this method also as a source-to-source transformation, which will allow us to compare the state-spaces generated by the exhaustive classical method to the one proposed here.

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